

HOW TO USE "MODERN" LOG TABLESIntro

1. Mathematics is the foundation of gunnery. This subj is taught to the reqr degree in all gunnery courses run by the School of Arty. Log table is a simplified way to solve gunnery problems, though with the intro of cal and com the use of such table has reduced a lot, yet the knowledge on log table forms the basics of understanding of mathematics for a student. The explanation of log table given in MOFG pamp is not elaborate and hence students are often found to avoid log problems. This handout explains log table in a simple manner so that students, who are due to come for different courses in the School of Arty, can have a clear idea about logarithm.

The Base

2. First of all, thinking about powers of 10, we can write

10^1	=	10
10^2	=	100
10^3	=	1000
10^4	=	10000
10^5	=	100000
10^6	=	1000000

and so on...

Now, if we multiply ten thousand by one hundred we get one million, or writing this down:-

$$10,000 \times 100 = 1,000,000$$

Or

$$10^4 \times 10^2 = 10^6 \text{ (Notice } 4 + 2 = 6\text{)}$$

In other words, we can **multiply** two numbers by **adding** their powers. These powers are the **logarithms** of the numbers. For instance, the logarithm of 100 is 2.

This is the modern way of describing logarithms; they are **powers** of a particular **base**. In the case we are looking at, the base is 10. (Sometimes written Log_{10}). So we can write $\text{Log}_{10}100=2$.

3. What about the logarithm of 101 or 15.27?

This is where the tables come in, They contain all the intermediate values to fill in the gaps between 10 and 100 and 100 and 1000 and so on.

To make the tables a manageable size, they do not contain the logarithms of all possible numbers, instead, the logarithm is split into two parts, separating the size of the number from its value. For example:-

Considering the numbers 15, 150 and 1500

15 lies between 10 and 100

so its logarithm must be between 1 and 2, ie it is 1.

something

150 lies between 100 and 1000

so its logarithm must be between 2 and 3, ie it is 2.

something

1500 lies between 1000 and 10000

so its logarithm must be between 3 and 4, ie it is 3.

something

In each case the ".*something*" has the same value and this is what is printed in the tables. It is known as the *MANTISSA*. You have to supply the number (1,2,3 ...) that goes before the decimal point. It is known as the **Characteristic**. In simpler words, for any log, the number to the left of the decimal point is called the **characteristic**, and the number to the right of the decimal point is called the **mantissa**. The characteristic only locates the decimal point of the number.

In the tables, the logarithm of 15 is printed as 1761, so for our examples,

The logarithm of 15 is 1.1761

The logarithm of 150 is 2.1761

The logarithm of 1500 is 3.1761 and so on...

4. The table below is an extract from a book of 4 figure log tables. Note that all the logarithms are decimal fractions, but the decimal point is not printed. This is a long standing tradition. Differences refer to the least significant figures (rightmost) and so should have zeros in front of them, again these are understood to be there, they are not printed. This does, in fact, make the tables more readable but takes some getting used to. To look up at logarithm of say, 15.27, you would

- First work out the characteristic, in this case it is 1
- Run your index finger down the left-hand column until it reaches 15
- Now move it right until it is on column 2 (it should be over 1818)
- Using another finger, find the difference on column 7 of the differences (20)
- Add the difference. So the logarithm of 15.27 is $1.1818 + 0.0020 = 1.1838$

LOGARITHMS											— Mean Differences —								
0	1	2	3	4	5	6	7	8	9		123	4	5	6	7	8	9		
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	48	12	17	21	25	29	33	37	
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	48	11	15	19	23	28	30	34	
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	37	10	14	17	21	24	28	31	
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	36	10	13	18	19	23	26	29	
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	36	9	12	15	18	21	24	27	
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	36	8	11	14	17	20	22	25	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	123	4	5	8	7	7	8		
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	123	4	5	5	6	7	8		
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	123	4	4	5	6	7	8		
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	123	4	4	5	6	7	8		
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	123	3	4	5	6	7	8		

Calculations Involving Logarithms

5. Because logarithms are exponents, mathematical operations involving them follow the same rules as those for exponents.

Some Examples Using Log Tables

6. **Multiply 15.27 by 48.54**

a. Look up the logarithms as before, the log of 15.27 is 1.1838 and the log of 48.54 is 1.6861

Add the logarithms

$$\begin{array}{r} 1.1838 \\ + 1.6861 \\ \hline 2.8699 \end{array}$$

At this point we have the logarithm of the answer - so what now?

It is possible to use the log tables backwards, but most people would have turned to the next page for the table of antilogarithms - printed below.

An extract from a 1966 table of anti-logarithms																			
ANTILOGARITHMS											— Mean Differences —								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5178	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5348	5358	1	2	4	5	8	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	8	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
.86	7244	7261	7278	7295	7311	7328	7345	7382	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7782	7780	7798	7816	7814	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	13	17

Mentally remove and store the characteristic (2)

Run index finger down the left-hand column until it finds **.86**

Move index finger along the row until it is on column 9 (it should now be over 7396)

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Move other finger to difference column 9 and read the difference (15)

Add the difference.

$$\begin{array}{r} 7396 \\ + 15 \\ \hline 7411 \end{array}$$

The characteristic was 2, meaning the answer is between 100 and 1000 so ...

Insert the decimal point in the correct place **741.1**

Try this on a calculator - the answer is different, 741.2. This is often the case, because of rounding errors. Log tables were faster than hand methods but four figure tables could only be relied on for three figure accuracy.

7. Divide 54.19 by 10.27

To divide we have to subtract the logarithms so:-

Look up the logarithms

Subtract them

$$\begin{array}{r} 1.7339 \\ - 1.0115 \\ \hline 0.7224 \end{array}$$

Then look up the antilog of .7224, (which is 5277)

Remembering that the characteristic is 0, insert the decimal point in the correct place **5.277**

This time the result agrees with a calculator!

8. You have learned various rules for manipulating and simplifying expressions with exponents, such as the rule that says that $x^3 \times x^5$ equals x^8 because you can add the exponents. There are similar rules for logarithms.

Log Rules:

- a. $\log_b(mn) = \log_b(m) + \log_b(n),$ $\log xy = \log x + \log y$
- b. $\log_b(m/n) = \log_b(m) - \log_b(n)$ $\log x/y = \log x - \log y$
- c. $\log_b(m^n) = n \cdot \log_b(m)$ $\log x^y = y \log x$



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d. $\log_b^n m = \log m^{1/n} = 1/n \log m$ $\log \sqrt[n]{x} = \log x^{1/n} = (1/n) \log x$

In less formal terms, the log rules might be expressed as:

- (1) Multiplication inside the log can be turned into addition outside the log, and vice versa.
- (2) Division inside the log can be turned into subtraction outside the log, and vice versa.
- (3) An exponent on everything inside a log can be moved out front as a multiplier, and vice versa.

Warning: Just as when you're dealing with exponents, the above rules work only if the bases are the same. For instance, the expression " $\log_a(m) + \log_b(n)$ " cannot be simplified, because the bases (the "a" and the "b") are not the same, just as $x^2 \times y^3$ cannot be simplified (because the bases x and y are not the same).

9. Find the Square Root of 52.73. Square roots are formed by dividing the logarithm by 2 (for cube roots, divide by 3 ... etc). Remember the formula : $\log \sqrt[n]{x} = \log x^{1/n} = (1/n) \log x$

- a. Look up the logarithm of 52.73 (1.7220)
- b. Divide by 2. ($1.7220 \div 2 = 0.8610$)
- c. Look up the antilog of .8610 (7.261)
- d. Remembering that the characteristic is 0, Insert the decimal point in the correct place 7.261

This time the calculator gives 7.2615, so the result should be 7.262

10. $\log 5.0 \times 10^6 = \log 5.0 + \log 10^6 = 0.70 + 6 = 6.70$

11. $\log (154/25) = \log 154 - \log 25 = 2.188 - 1.40 = 0.788 = 0.79$

12. $\log (5.46 \times 10^{-3})^6 = 6 \log 5.46 \times 10^{-3} = 6 \times (-2.263) = -13.58$

13. $\log (4.35)^{1/4} = 1/4 \log 4.35 = 0.160$

Quiz: Give the correct answer to the correct number of significant figures.

Question 1 $\log 23.0 = ?$

Question 2 Find x if $\log x = 10.23$.

Question 3 $\log (1.2 \times 10^6)^3 = ?$

Answers: (1) 1.362 (2) 1.7×10^{10} (3) 18.24

Expanding Logarithms

Log rules can be used to simplify expressions, to "expand" expressions, or to solve for values.

- **Expand $\log_3(2x)$.**

When they say to "expand", they mean that they've given you one log expression with lots of stuff inside it, and they want you to use the log rules to take the log apart into lots of separate logs, each with only one thing inside. That is, they've given you one log with a complicated argument, and they want you to convert this to many logs, each with a simple argument.

I have a "2x" inside the log. Since "2x" is multiplication, I can take this expression apart and turn it into an addition outside the log:

$$\log_3(2x) = \log_3(2) + \log_3(x)$$

The answer they are looking for is:

$$\log_3(2) + \log_3(x)$$

Do not try to evaluate " $\log_3(2)$ " in your calculator. While you would be correct in saying that " $\log_3(2)$ " is just a number, they're actually looking here for the "exact" form of the log, as shown above, and not a decimal approximation from your calculator.

- **Expand $\log_4(16/x)$.**

I have division inside the log, which can be split apart as subtraction outside the log, so:

$$\log_4(16/x) = \log_4(16) - \log_4(x)$$

The first term on the right-hand side of the above equation can be simplified to an exact value, by applying the basic definition of what a logarithm is:

$$\log_4(16) = 2$$

Then the original expression expands fully as:

$$\log_4(16/x) = 2 - \log_4(x)$$

Always remember to take the time to check to see if any of the terms in your expansion (such as the $\log_4(16)$ above) can be simplified.

- **Expand $\log_5(x^3)$.** Copyright © Elizabeth Stapel 2006-2008 All Rights Reserved

The exponent inside the log can be taken out front as a multiplier:

$$\log_5(x^3) = 3 \cdot \log_5(x) = 3\log_5(x)$$

- Expand the following:

$$\log_2 \left(\frac{8x^4}{5} \right)$$

The 5 is divided into the $8x^4$, so split the numerator and denominator by using subtraction:

$$\log_2 \left(\frac{8x^4}{5} \right) = \log_2(8x^4) - \log_2(5)$$

Don't take the exponent out front yet; it is only on the x , not the 8, and you can only take the exponent out front if it is "on" everything inside the log. The 8 is multiplied onto the x^4 , so split the factors by using addition:

$$\log_2(8x^4) - \log_2(5) = \log_2(8) + \log_2(x^4) - \log_2(5)$$

The x has an exponent (which is now "on" everything inside its log), so move the exponent out front as a multiplier:

$$\begin{aligned} \log_2(8) + \log_2(x^4) - \log_2(5) \\ = \log_2(8) + 4\log_2(x) - \log_2(5) \end{aligned}$$

Since 8 is a power of 2, I can simplify the first log to an exact value:

$$\begin{aligned} \log_2(8) + 4\log_2(x) - \log_2(5) \\ = 3 + 4\log_2(x) - \log_2(5) \end{aligned}$$

Each log contains only one thing, so this is fully simplified. The answer is:

$$3 + 4\log_2(x) - \log_2(5)$$

- Expand the following:

$$\log_3 \left(\frac{4(x-5)^2}{x^4(x-1)^3} \right)$$

Use the log rules, and don't try to do too much in one step:

$$\begin{aligned}
 \log_3 \left(\frac{4(x-5)^2}{x^4(x-1)^3} \right) &= \log_3 \left(4(x-5)^2 \right) - \log_3 \left(x^4(x-1)^3 \right) \\
 &= \left[\log_3(4) + \log_3((x-5)^2) \right] - \left[\log_3(x^4) + \log_3((x-1)^3) \right] \\
 &= \log_3(4) + \log_3((x-5)^2) - \log_3(x^4) - \log_3((x-1)^3) \\
 &= \log_3(4) + 2\log_3(x-5) - 4\log_3(x) - 3\log_3(x-1)
 \end{aligned}$$

Then the final answer is: Copyright © Elizabeth Stapel 1999-2009 All Rights Reserved

$$\log_3(4) + 2\log_3(x-5) - 4\log_3(x) - 3\log_3(x-1)$$

SIMPLIFYING LOGARITHMS

The logs rules work "backwards", so you can simplify ("compress"?) log expressions. When they tell you to "simplify" a log expression, this usually means they will have given you lots of log terms, each containing a simple argument, and they want you to combine everything into one log with a complicated argument. "Simplifying" in this context usually means the opposite of "expanding".

- Simplify $\log_2(x) + \log_2(y)$.

Since these logs have the same base, the addition outside can be turned into multiplication inside:

$$\log_2(x) + \log_2(y) = \log_2(xy)$$

The answer is $\log_2(xy)$.

- Simplify $\log_3(4) - \log_3(5)$.

Since these logs have the same base, the subtraction outside can be turned into division inside:

$$\log_3(4) - \log_3(5) = \log_3(4/5)$$

The answer is $\log_3(4/5)$.

- Simplify $2\log_3(x)$.

The multiplier out front can be taken inside as an exponent:

$$2\log_3(x) = \log_3(x^2)$$

- Simplify $3\log_2(x) - 4\log_2(x+3) + \log_2(y)$.

I will get rid of the multipliers by moving them inside as powers:

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$$3\log_2(x) - 4\log_2(x + 3) + \log_2(y) \\ = \log_2(x^3) - \log_2((x + 3)^4) + \log_2(y)$$

Then I'll put the added terms together, and convert the addition to multiplication:

$$\log_2(x^3) - \log_2((x + 3)^4) + \log_2(y) \\ = \log_2(x^3) + \log_2(y) - \log_2((x + 3)^4) \\ = \log_2(x^3y) - \log_2((x + 3)^4)$$

Then I'll account for the subtracted term by combining it inside with division:

$$\log_2(x^3y) - \log_2((x + 3)^4) = \log_2\left(\frac{x^3y}{(x + 3)^4}\right)$$

- Let $\log_b(2) = 0.3869$, $\log_b(3) = 0.6131$, and $\log_b(5) = 0.8982$. Using these values, evaluate $\log_b(10)$.

Since $10 = 2 \times 5$, then:

$$\log_b(10) = \log_b(2 \times 5) = \log_b(2) + \log_b(5)$$

Since I have the values for $\log_b(2)$ and $\log_b(5)$, I can evaluate:

$$\log_b(2) + \log_b(5) = 0.3869 + 0.8982 = 1.2851$$

Then $\log_b(10) = 1.2851$.

- Let $\log_b(2) = 0.3869$, $\log_b(3) = 0.6131$, and $\log_b(5) = 0.8982$. Using these values, evaluate $\log_b(9)$.

Since $9 = 3^2$, then:

$$\log_b(9) = \log_b(3^2) = 2\log_b(3)$$

Since I have the value for $\log_b(3)$, then I can evaluate:

$$2\log_b(3) = 2(0.6131) = 1.2262$$

Then $\log_b(9) = 1.2262$.

- Let $\log_b(2) = 0.3869$, $\log_b(3) = 0.6131$, and $\log_b(5) = 0.8982$. Using these values, evaluate $\log_b(7.5)$.

This one is a bit more complicated, but, after fiddling with the numbers for a bit, I notice that $7.5 = 15 \div 2$, so:

$$\log_b(7.5) = \log_b(15 \div 2) = \log_b(15) - \log_b(2)$$

And $15 = 5 \times 3$, so: Copyright © Elizabeth Stapel 2006-2008 All Rights Reserved

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$$\begin{aligned}\log_b(15) - \log_b(2) \\&= [\log_b(5) + \log_b(3)] - \log_b(2) \\&= \log_b(5) + \log_b(3) - \log_b(2)\end{aligned}$$

And now I can evaluate:

$$\begin{aligned}\log_b(5) + \log_b(3) - \log_b(2) \\&= 0.8982 + 0.6131 - 0.3869 \\&= 1.1244\end{aligned}$$

Then $\log_b(7.5) = 1.1244$.

- Let $\log_b(2) = 0.3869$, $\log_b(3) = 0.6131$, and $\log_b(5) = 0.8982$. Using these values, evaluate $\log_b(6)$.

Since $6 = 2 \times 3$, then:

$$\log_b(6) = \log_b(2 \times 3) = \log_b(2) + \log_b(3)$$

Since I have these values, I can evaluate:

$$\log_b(2) + \log_b(3) = 0.3869 + 0.6131 = 1.0000$$

Then $\log_b(6) = 1.0000$.

NEGATIVE LOGARITHMS

Definition

Logarithms of be less than unity are termed as the negative logarithms or those logarithms which have negative characteristics.

Example.

Find log of .4792 (.4792 is less than a unit)

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$$\begin{aligned}\text{Log } .4792 &= \log \frac{4792}{10000} && (\text{removal of decimal}) \\ &= \log 4792 - \log 10,000 && (\text{by application of law of quotient}). \\ &= 3.6804 - \log 10^4 = 3.6804 - 4 \\ \therefore \log .4792 &= -.3196 \dots\dots\dots A.\end{aligned}$$

It is essential to convert this negative mantissa into positive mantissa into positive mantissa. To achieve this “+1” has been added in and “(-)” has been subtracted from equation “A”. $\log .4792 = -.3196 + 1 - 1$ (equation remains unchanged)
 $= (1-.3196) - 1$
 $= .6804 - 1$ (Positive mantissa and negative characteristics)
 $= 1.6804$ (“(-)” is written as bar one).
Therefore all log tables have been constructed to provide positive mantissa.

Prac Problem 1

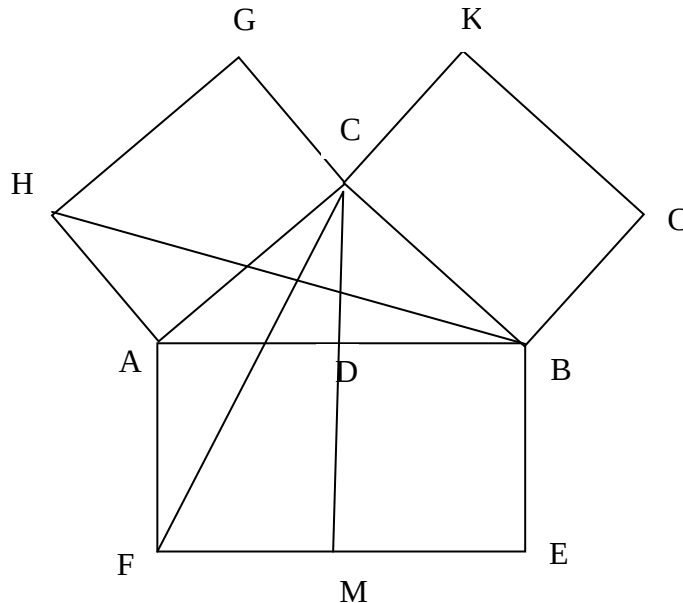
Log .03

ans : 2.4771

PYTHAGORAS THEOREM

□ The Theorem is that in a Right Angle Triangle the Sum of the Squares on Two Sides is Equal to the Square on the Hypotenuse.

❖ GIVEN : ABC is a Triangle With Angle CAB The right Angle.



Prove. $\triangle ACG$, $\triangle ACB$ and $\triangle BCK$ are RT angle

ACK and BCG are st lines

Now consider $\triangle ABH$ and $\triangle AFC$

$AB=AF$ and $AH=AC$ (sides of same sq)

$\angle BAH = \angle FAC$ (each being equal to $90^\circ + \angle BAC$)

$\triangle ABH \cong \triangle AFC$

$\triangle BAH$ and $\triangle ACH$ stand on the same base AH and are between the same $\parallel$ AH and BG

$\text{sq } ACGH = 2 \triangle BAH$ ----- (i)

For similar reasons rectangle $ADFM = 2 \triangle CAF$ (being on the same AF and between the same.

$\parallel$ AF and CM)

SINCE $\triangle BAH = \triangle CAF$ (already proved)

$\text{sq } ACGH = \text{RECTANGLE } ADFM$.

Similarly by drawing AL and CE it can be proved that sq.

$BCKL = \text{rectangle } BDME$ ----- (ii)

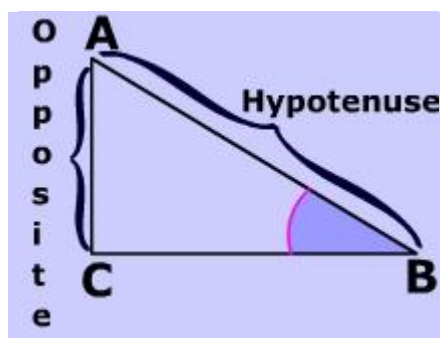
By adding equations (i) and (ii)

$\text{sq } ACGH + \text{sq } BCKL = \text{rectangle } ADFM + \text{rectangle } BDME = \text{sq } ABEF$

Hence the theorem is proved.

Sine Function

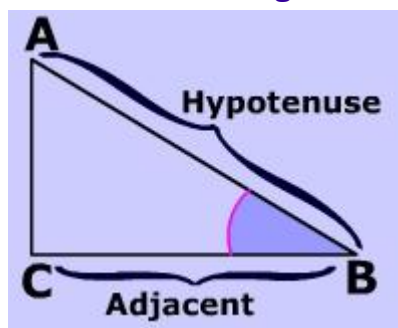
The **sine** of an angle is always the ratio of the (opposite side)/(hypotenuse).



Sine of $\angle ABC = AC / AB$
(opposite side)/(hypotenuse)

Cosine Function

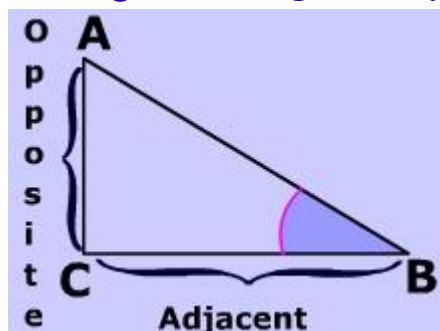
The **cosine** of an angle is always the ratio of the (adjacent side/ hypotenuse)



The cosine of $\angle ABC = CB / AB$
(adjacent side)/(hypotenuse)

Tangent Function

The **tangent** of an angle is always the ratio of the (opposite side)/(adjacent side)



The tangent of $\angle ABC = AC / CB$

Double Angle Formula

How to use formula to express exact values:

The double angle identities take two different formulas:

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

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Angle Sum formula

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

:

These formulas allow you to express the exact value of trigonometric expressions that you could not otherwise express. Consider the $\sin(105^\circ)$. Unlike the $\sin(30)$ which can be expressed as $\frac{1}{2}$, the $\sin(105)$ cannot simply be represented as a rational expression. However, the angle sum formula allows you to represent the exact value of this function:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(60 + 45) = \sin 60 \cos 45 + \cos 60 \sin 45$$

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$$\frac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}}{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

Angle Difference formula

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

These formulas allow you to express the exact value of trigonometric expressions that you could not otherwise express. Consider the $\cos(15^\circ)$. Unlike the $\cos(60^\circ)$ which can be expressed as $\frac{1}{2}$, the $\cos(15^\circ)$ cannot simply be represented as a rational expression. However, the angle difference formula allows you to represent the exact value of this function:

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\cos(45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30$$

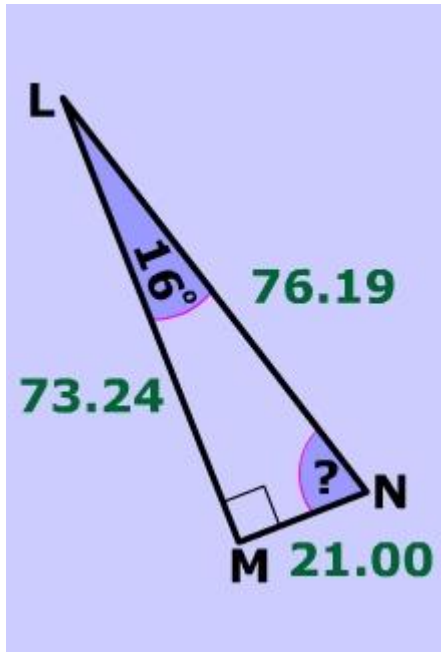
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$$\frac{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}}{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

INVERSE SINE, COSINE AND TANGENT

The inverse of SOHCAHTOA

The inverse trigonometric functions ($\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$) allow you to find the measure of an angle in a right triangle. All that you need to know are any two sides as well as how to use SOHCAHTOA.



Vs. Interior Angle Sum

Compare This Method To the tried and true theorem that the sum of the interior **angles** of a **triangle** is 180° .

What is the degree measure of $\angle LNM$?

Interior Angle Sum Way

Since the total measure of the **interior angles** of a **triangle** is 180 degrees we can verify the measure of $\angle LNM$

$$\Rightarrow 180^\circ - 16^\circ - 90^\circ = 74^\circ$$

Alternately, you could use the inverse of one of the **SOHCAHTOA** functions, in this case the inverse of sine ($\sin^{-1}$)! To find, an **angle** of a **right triangle** all that we need to know is the length of two sides! Then use the same

SOHCAHTOA ratios --just in a different fashion See the example below.

Inverse SOHCAHTOA Way

$$\sin^{-1}(73.24/76.19) = 74^\circ$$

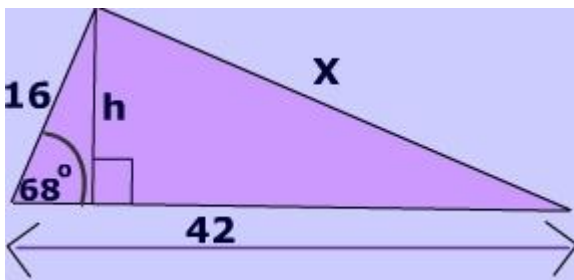
THE AREA OF A TRIANGLE: SAS FORMULA

The Side Angle Side Formula

The **Side Angle Side formula** for finding the area of a triangle is a way to use the sine trigonometric function to calculate the height of a triangle and use that value to find the area of the triangle.

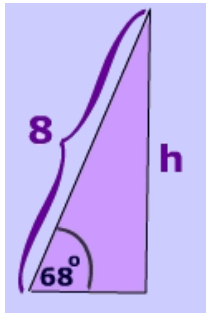
The SAS formula = $\frac{1}{2} (\text{side}_1 \times \text{side}_2) \times \sin(\text{included angle})$

$$\text{The Side Angle Side Formula} = \frac{1}{2} (42 \times 16) \times \sin(68^\circ)$$



The Side Angle Side formula simply is an application of the [sine function](#) to find the height. For more information, read below.

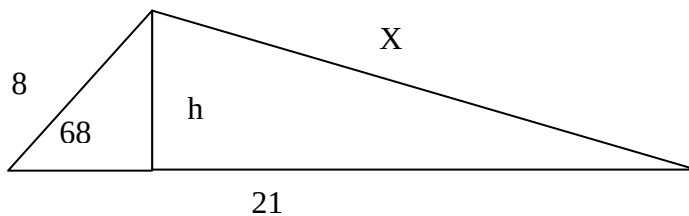
To use the [Side Angle Side Formula](#), make sure that you are proficient using [SOHCAHTOA](#). Specifically, you should be able to [solve problems like the one immediately](#) below. The SAS formula is simply an application of sine to find the height of a triangle and then to use that height to find the triangle's area.



In the triangle on the left, how long is h?

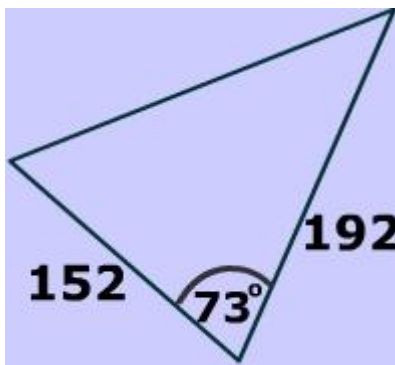
- Length of H
- ☐ $\sin(68) = \text{opposite/hypotenuse}$
 - ☐ $\sin(68) = h/8$
 - ☐ $h = 8 \cdot \sin(68) = 7.4$

1. What is the height of the triangle below?



Side Angle Side Formula

Practice Problems

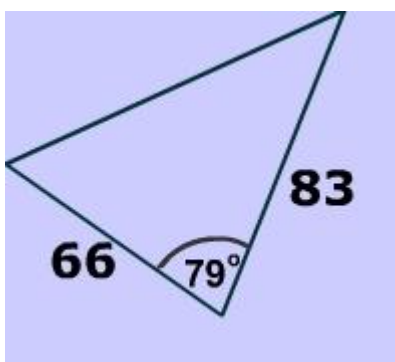


Use the [Side Angle Side formula](#) to find the [area](#) of the [triangle](#) on the left.

Area

$$\text{Area} = \frac{1}{2}(152 \times 192(\sin 73^\circ)) = 13,954$$

Use [side angle side formula](#) to find the [area](#) of the [triangle](#) on the left.



Area

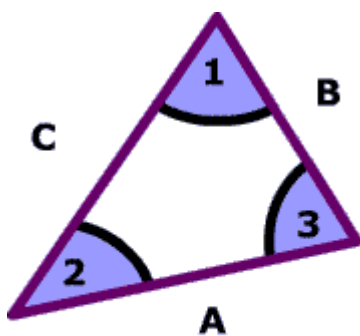
$$\text{Area} = \frac{1}{2}(66 \times 83(\sin 79^\circ)) = 2,689$$

THE LAW OF COSINES

The law of cosines is a formula that relates the three sides of a triangle to the cosine of a given angle. This formula allows you

- to calculate the side length of non-right triangle as long as you know two sides and an angle
- to calculate any angle of a triangle if you know all three side lengths

Laws of Cosine Formula:



The formula for the law of cosines expresses the following relationships between the sides angles of any triangle:

- ☐ $A^2 = B^2 + C^2 - 2(B)(C)\cos(\angle 1)$
- ☐ $B^2 = A^2 + C^2 - 2(A)(C)\cos(\angle 2)$
- ☐ $C^2 = A^2 + B^2 - 2(A)(B)\cos(\angle 3)$

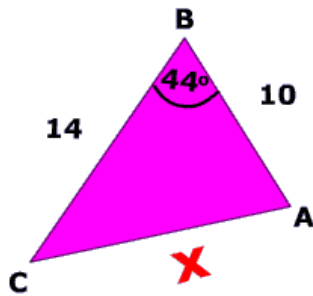
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The two common problems types that are suitable for the law of cosines are.

Problem type 1: when you know the lengths of 2 sides of a triangle and the angle in between the two sides

Problem type 2: when you know the lengths of three sides of a triangle and want to know a particular angle

Example of Law of Cosines Formula



$$\square x^2 = 10^2 + 14^2 - 2(10)(14)\cos(44^\circ)$$

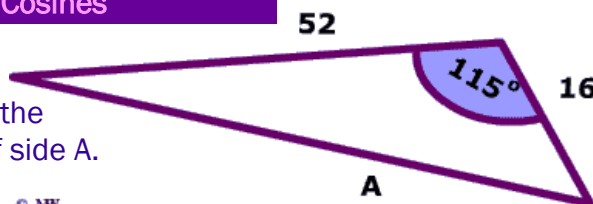
$$\square x^2 = 296 - 280\cos(44)$$

$$\square x^2 = 94.6$$

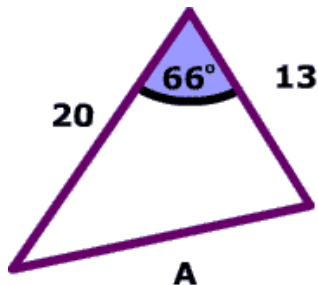
$$\square x = \sqrt{94.6} = 9.73$$

Practice applying Law of Cosines

Practice Problem 1) Use the law of cosines formula to calculate the length of side A.



law of cosines formula



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Answer

$$\square A^2 = 20^2 + 13^2 - 2(20)(13)\cos(66)$$

$$\square A^2 = 400 + 169 - 520\cos(66)$$

$$\square A^2 = 569 - 211$$

$$\square A^2 = 358 \rightarrow A = \sqrt{358} = 18.9$$

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Practice Problem 2) Use the law of cosines formula to calculate the length of side A.

Step 1: set up the formula

Step 2: do the calculation

$$\square A^2 = 52^2 + 16^2 - 2(52)(16)\cos(115^\circ)$$

$$\square A^2 = 2,704 + 256 - 1,664\cos(115)$$

$$\square A^2 = 2,960 - 1,664(-.423)$$

$$\square A^2 = 2,960 + 703 = 3663$$

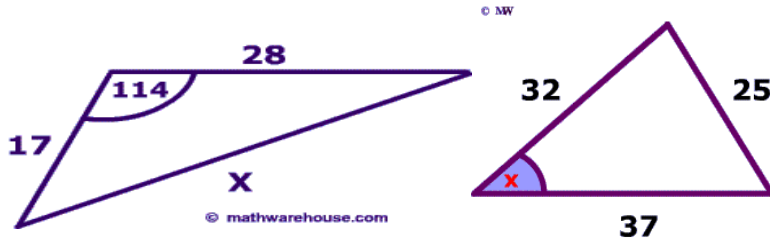
$$\square A = \sqrt{3663} = 60.5$$

$$x^2 = 28^2 + 17^2 - 2(17)(28)\cos 114^\circ$$

$$x^2 = 28^2 + 17^2 - 2(17)(28)\cos 114^\circ$$

$$x^2 = 1460 \rightarrow x = \sqrt{1460} = 38.2$$

Practice Problem 3) Use the law of cosines to calculate X



Step 1: set up formula

Solve the equation

Practice applying Law of Cosines

law of cosines to find an angle

Practice Problem 4) Use the law of cosines to calculate x.

$$25^2 = 32^2 + 37^2 - 2(32)(37)\cos(x)$$

$$625 = 2393 - 2,368 \cos(x)$$

The law of Cosines and the pythagorean theorem.

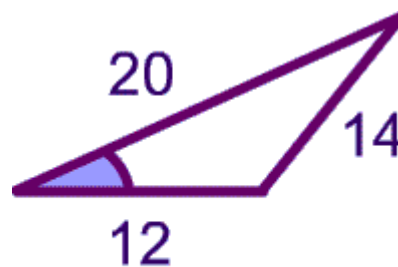
Use the law of cosines to calculate the value of x. This exercise should help you see the connection between the law of cosines and the pythagorean theorem.

Practice Problem 5) Use the law of cosines to calculate the measure of the shaded anlge.

$$-1760 = - 2,368 \cos(x)$$

$$.7432 = \cos x$$

$$\cos^{-1}(.7432) = 42.0^\circ$$



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$$\square 14^2 = 20^2 + 12^2$$

$$-2(12)(20)\cos(\text{angle})$$

Solve the equation

☐ Solve the equation in terms of $\cos(\text{angle})$ and then use $\cos^{-1}$ to determine the angle

measurement

$$14^2 - 20^2 - 12^2 = (20)(12) \cos(\text{angle})$$

$$-348 = -480 \cos(\text{angle})$$

$$.675 = \cos(\text{angle})$$

$$\text{angle} = \cos^{-1}(.675) = 47.5$$

QUESTION AND ANS - MATH

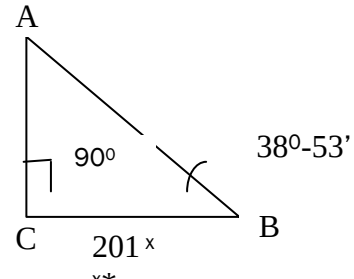
1. ABC is a triangle whose $\angle ACB = 90^\circ$, $\angle ABC = 38^\circ-53'$ side BC = 201^x, Find out side AB & AC.

$$\frac{P}{B} = \tan \angle B$$

$$AC = 162.09^x$$

$$\frac{P}{H} = \sin \angle B, AB = 258.21^x$$

Ans: AC = 162.09^x and AB = 258.21^x



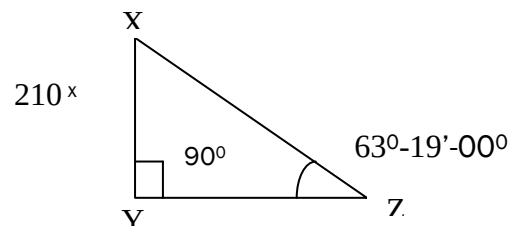
2. XYZ is a triangle, $\angle XYZ$ is 90° . $\angle XZY$ is $63^\circ-19'$, Side XY is 210^x. Find out side XZ and YZ.

$$\frac{P}{H} = \sin \angle Z$$

$$XZ = 235.03^x$$

$$\frac{P}{B} = \tan \angle Z, YZ = 105.54^x$$

Ans: XZ = 235.03^x and YZ = 105.54^x.



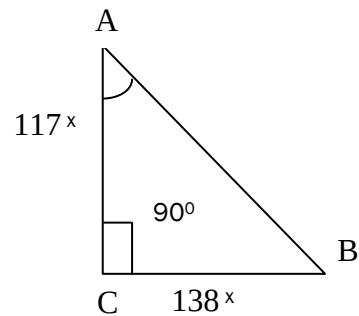
3. ABC is a triangle, $\angle ACB$ is 90° . Side AC is 117^x and side BC 138^x. Find out side $\angle BAC$ and side AB.

$$\frac{P}{B} = \tan \angle A$$

$$\tan \angle A = 49^\circ - 42' - 28.27''$$

$$\frac{B}{H} = \cos \angle A, AB = 180.92^x$$

Ans: $\angle BAC = 49^\circ - 42' - 28.27''$ and $AB = 180.92^x$.



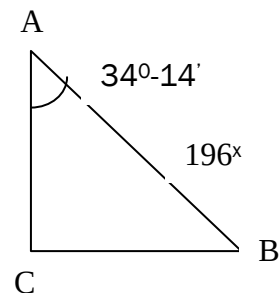
4. ABC is a triangle, $\angle ACB$ is 90° . $\angle BAC$ is $34^\circ-14'$, side AB 196^x. Find out side AC and side BC.

$$\frac{B}{H} = \cos \angle A$$

$$AC = 162.04^x$$

$$\frac{P}{H} = \sin \angle A, BC = 110.26^x$$

Ans: Side AC = 162.04^x and side BC = 110.26^x.

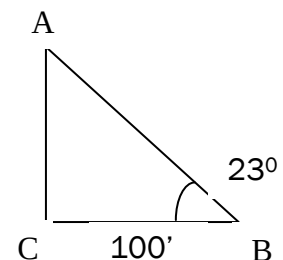


5. A man standing 100' away from a wall, he measured the A/S to the wall 23° . Find out the ht of the wall.

$$\frac{P}{B} = \tan \angle B$$

$$AC = 42.45'$$

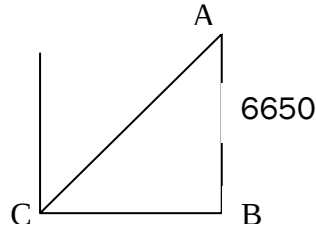
Ans: AC = 42.45'.



6. While engaging tgt P 4001. No 2 gun was the ranging. The fired range to the tgt from no 2 gun 6650^x. Distance from gun no 2 to gun no 4 is 60^x. Find out the conc for no 4 gun.

$$\frac{P}{B} = \tan \angle B$$

Ans: Tgt = 00°-31'-29.4" .



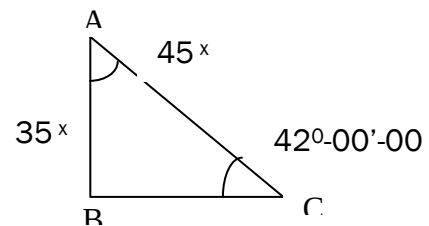
7. ABC is a triangle, $\angle ACB$ is 42°-00'. Side AB is 35^x and side AC is 45^x. Find out $\angle BAC$ and side BC.

$$\frac{P}{B} = \tan \angle C$$

$$BC = 38.87^x$$

$$\frac{P}{B} = \tan \angle A, \angle A = 48^{\circ}-00'$$

Ans: $BC = 38.87^x$, $\angle A = 48^{\circ}-00'$.

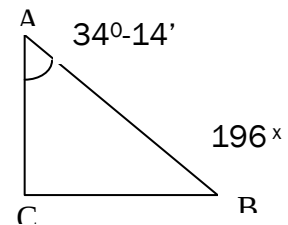


8. ABC is a triangle, $\angle ACB$ is 90°. $\angle BAC$ is 34°-14' side AB is 196^x. Find out side AC & BC.

$$\frac{B}{H} = \cos \angle A, AC = 164.04^x$$

$$\frac{P}{H} = \sin \angle A, BC = 110.26^x$$

Ans: $AC = 164.04^x$, and $BC = 110.26^x$.

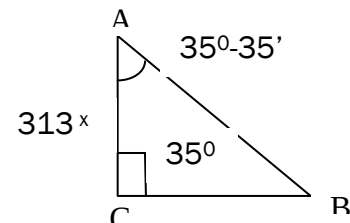


9. ABC is a triangle, $\angle ACB$ is 90° side AC is 313^x $\angle BAC$ 35°-35'. Find out side AB & $\angle ABC$.

$$\frac{B}{H} = \cos A, AB = 384.87^x \text{ Ans.}$$

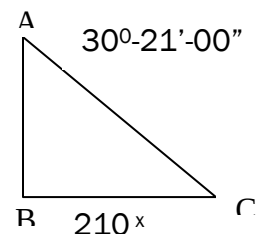
$$\frac{P}{H} = \sin B, B = 54^{\circ}-25' \text{ Ans.}$$

Ans: Side $AB = 384.87^x$ and $\angle ABC = 54^{\circ}-25'$.



10. ABC is a Rt angle triangle, side BC is 210 yds and angle BAC is 30°-21'-00". Find out side AB and AC (angle ABC is 90°).

$$\frac{B}{H} = \tan \angle A, P = B \times \tan AB: P = 358.65^x.$$



$$\frac{P}{H} = \sin \angle A, H = 415^{\circ}.16^x.$$

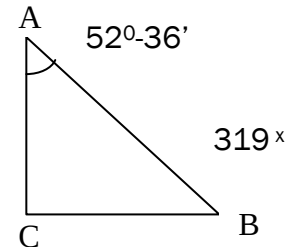
Ans: Side AB is 358.65^x and AC is 415.61^x .

11. ABC is a rt angle triangle side AC is 319^x yds and angle BAC is $52^{\circ}-36'$. Find out side AB and BC. (angle C is 90°).

$$\frac{b}{H} = \cos \angle A, \frac{P}{H} = \sin \angle A$$

$$H = \frac{b}{\cos \angle A}, P = H \sin \angle A.$$

$$H = 525.21^x, P = 417.23^x$$



Ans: Side BC is 417.23^x and side AB is 525.21^x .

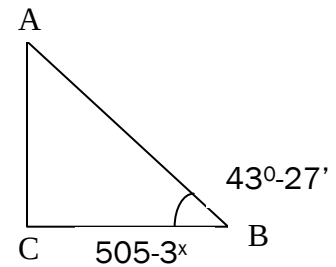
12. ABC is a rt angle triangle side BC is 505.3 yds and angle ACB is $43^{\circ}-27'$. Find out side AB and AC ($\angle BAC = 90^{\circ}$).

$$\frac{P}{H} = \sin \angle C, \frac{B}{H} = \cos \angle C$$

$$P = H \sin \angle C, H = B \div \cos \angle B.$$

$$P = 478.67^x \text{ AB } H = 696.33^x \text{ AC}$$

$$\frac{P}{H} = \sin \angle A \text{ or } \sin \angle A = P \div H, \sin \angle A = 46^{\circ}-32'-59.9''$$



Ans: Side AB is 478.67^x AC is 696.03 and $\angle A$ is $46^{\circ} 32' - 59.9''$.

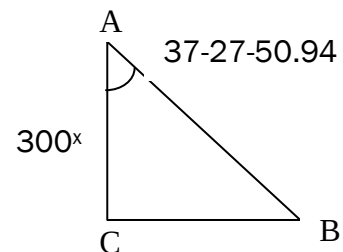
13. ABC is a RT Angle. Side AB is 300 yds and BC is 229.9 yds. Find out $\angle BAC$ and $\angle ACB$.

$$\frac{P}{B} = \tan \angle A, \frac{P}{B} = \tan \angle C$$

$$\text{or } \frac{BC}{AB} = \tan \angle A, \tan \angle C = \frac{AB}{BC}$$

$$\tan \angle A = P \div B, \tan \angle C = AB \div B$$

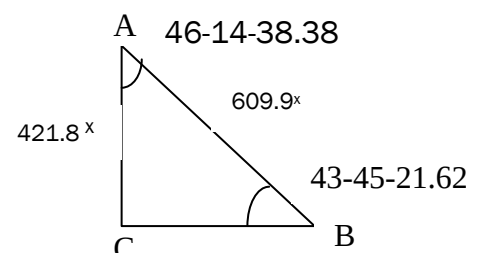
$$\angle A = 37^{\circ}27'-50.94^{\circ}, \tan \angle C = 52^{\circ}-32'-9.06''$$



Ans: $\angle BAC = 37^{\circ}-27'-50.94''$ and $\angle ACB = 52^{\circ}-32'-6.06''$.

14. ABC is a RT Angle triangle. Side AC is 609.9 yds and AB is 421.8 yds. Find out Angle $\angle BAC$ and $\angle ACB$.

$$\frac{P}{H} = \sin \angle C, \frac{B}{H} = \cos \angle A$$



$$\sin \angle C = \frac{AB}{AC} \quad \cos \angle A = \frac{AB}{AC}$$

$$\sin \angle C = AB \div AC \quad \cos \angle A = AB \div AC$$

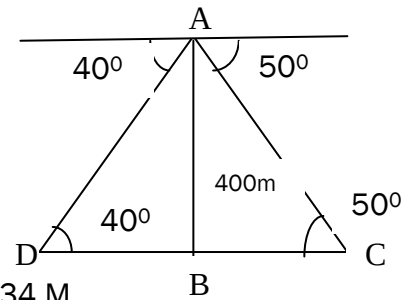
$$\angle C = 43^{\circ}45'21.62'' \quad \angle A = 46^{\circ}14'38.38''$$

Ans: $\angle BAC = 46^{\circ}14'38.38''$ and $\angle ACB = 52^{\circ}45'21.62''$.

15. Two ships were approaching towards a light house from opposite direction. The angle of depression to those two ships as measured from the top of the light house were 50° and 40° respectively. If the height of the house is 400m. Find out the distance between the two ships at the time measuring the angle.

$$\tan \angle C = \frac{P}{B}, \text{ or } B = \frac{P}{\tan \angle C}, BC = 335.64 \text{ M}$$

$$\tan \angle D = \frac{P}{B}, \quad BD = 476.70 \text{ M}$$



Ans: The distance between two ships = $335.64 + 476.70 = 812.34 \text{ M}$

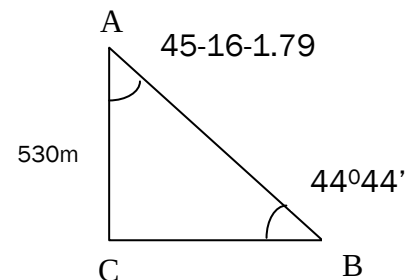
16. ABC is a right angle triangle whose $\angle ABC$ is 90° and $\angle BAC$ is $44^{\circ}44'$. If side AB is 530m, what will be the side AC, BC and $\angle BAC$?

$$\sin \angle B = \frac{P}{H}, \text{ or } P = H \times \sin \angle B$$

$$AC = 373.02 \text{ M}, BC = 376.51 \text{ M}$$

$$\tan \angle B = \frac{P}{B}, \text{ or } B = \frac{P}{\tan \angle B}$$

$$\sin \angle A = \frac{P}{H}, \angle BAC = 45^{\circ}16'1.79''$$



Ans: Side AC = 373.02 M, Side BC = 376.51 M & $\angle BAC = 45^{\circ}16'1.79''$.

17. A and B are two observers trying to measure the height of a pole which is dug into the earth. The vertical distance between two observers is 319 yds. A/S to the top of the pole is $20^{\circ}30'$ from upper observer and $43^{\circ}29'$ from lower observer. Find out the height of the pole.

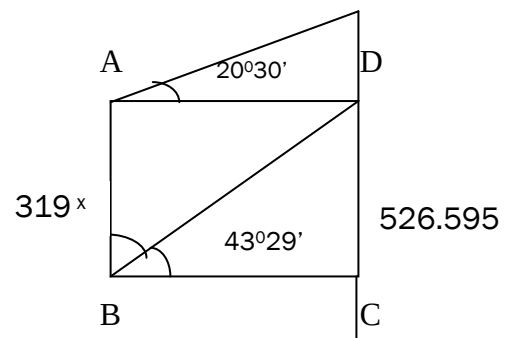
$\triangle ABD$,

$$\frac{a}{\sin \angle A} = \frac{d}{\sin \angle D}, \text{ or } a = d \times \sin \angle A + \sin \angle D$$

$$a = 765.2406556^x$$

$\triangle BCD$

$$\frac{P}{H} = \sin \angle B, \text{ or } P = H \times \sin \angle B$$



$$P = 526.5954116^x$$

$$526.5954116^x + 2$$

Ans: Ht of the pole = 528.5954116^x .

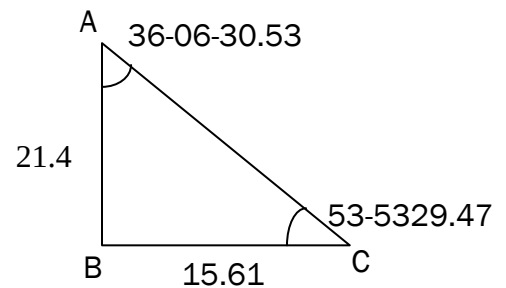
18. Two adjacent sides of a rectangle are 21.4 cm and 15.61 cm respectively . Find the angle which a diagonal of the rectangle makes with the side. Also find the length of the diagonal

$$\tan \angle C = \frac{21.4}{15.61} = 53^\circ-53'-29.47''$$

$$\tan \angle A = \frac{15.61}{21.4} = 36^\circ-06'-30.53''$$

$$\frac{BC}{AC} = \sin \angle A$$

$$AC = \frac{BC}{\sin \angle A} = 26.49 \text{ cm.}$$



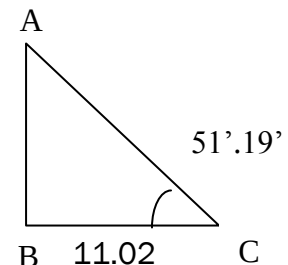
Ans: $\angle BCA = 53^\circ-53'-29.47''$ and $\angle BAC = 36^\circ-06'-30.53''$.

19. When the altitude of the sun is $51^\circ 19'$ find the ht of a flag staff whose shadow is 11.02m long.

$$\tan \angle C = \frac{AB}{BC}, \text{ or Side AB} = \tan \angle C \times BC$$

$$\text{Side AB} = 13.76 \text{ M}$$

Ans: Side AB = 13.76 m .



20. A balloon was released from a pt and rising vertically up. An observers standing some distance apart from that pt and observed the altitude and Bg 24° and 41° respectively. Another observer Loc 5 km due south of the other measured the same as $20^\circ 37'$ and 32° respectively. Find out the ht of the balloon.

$$\angle ACB = 90^\circ - 24^\circ = 66^\circ$$

$$\angle BAD = 180^\circ - 171 = 9^\circ$$

$\triangle ABD$

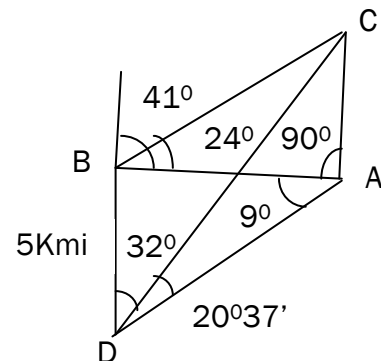
$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} \text{ or } b = \frac{a \sin \angle B}{\sin \angle A}$$

$$AD = 20.96913327 \text{ km}$$

$\triangle ADC$

$$\frac{P}{B} = \tan \angle D$$

$$P = \tan \angle D \times B, AC = 7.888741236 \text{ km}$$



△ ABD

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle D} \text{ or } d = \frac{ax \sin \angle D}{\sin \angle A} = AB = 16.93742054$$

$$\frac{P}{B} = \tan \angle B \text{ or } P = \tan \angle B \times B$$

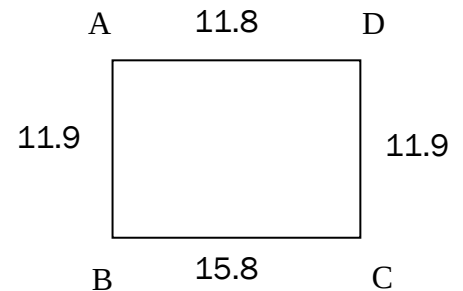
$$AC = 7.541025481 \text{ km}$$

Ans: AC = 7.541025481 km or 7.888741236 km

21. Two adjacent sides of a rectangle are 15.8 cms and 11.9 cms. Find the angle, which a diagonal of the rectangle makes with sides.

$$\tan \angle D = \frac{BC}{CD} = 53^{\circ} 0' 51.25''$$

$$\tan \angle B = \frac{CD}{BC} = 36^{\circ} 59' 8.75''$$



Ans: $\angle D = 53^{\circ} 0' 51.25''$ and $\angle B = 36^{\circ} 59' 8.75''$.

22. In the area of triangle is 100m and two of its sides are. 21m and 15m. Find the angle between these sides.

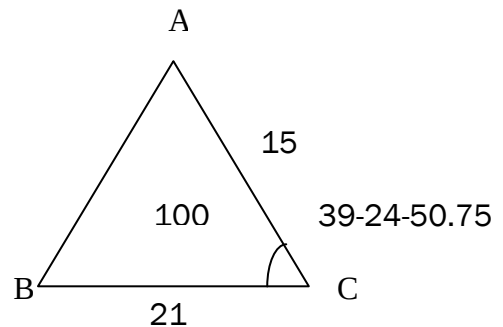
$$\triangle ABC = 100^m$$

$$\text{We know } S = \frac{1}{2} a \times b \times \sin \theta$$

$$= \frac{1}{2} \times c \times b \times \sin \angle A$$

$$100^m = \frac{1}{2} \times 15 \times 21 \times \sin \angle A$$

$$= 157.50 \times \sin \angle A$$



$$\sin \angle A = 100 \div 157.50 = 0.634920634 = 39^{\circ} 24' 50.75''$$

Ans: $39^{\circ} 24' 50.75''$

23. In the area of quadrilateral ABCD is which AB= 14.7cm BC= 9.6cm, CD =21.7cm and AD= 18.9cm and $\angle ABC = 137^{\circ}$

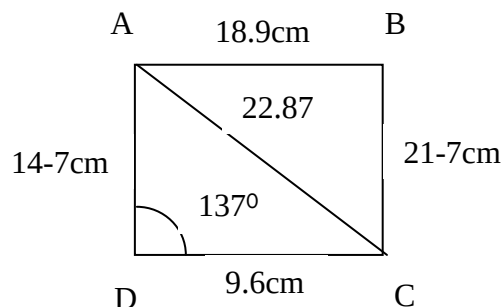
$$b = \sqrt{a^2 + c^2 - 2ac \cos \angle B}$$

$$b = \sqrt{522.85} \text{ cm} = 22.87 \text{ cm}$$

$$\triangle ABC \quad S = \frac{14.7 + 9.8 + 22.87}{2} = 23.68 \text{ cm}$$

$$\triangle ADC \quad S = \frac{18.9 + 21.7 + 22.87}{2} = 31.74 \text{ cm}$$

$$\text{Area of } \angle ABC = \sqrt{S(S-a)(S-b)(S-c)}$$



$$= \sqrt{23.68 (23.68-9.8) (23.68-22.87) (23.68-14.7)} = 48.90$$

$$\text{Area of } \angle ADC = \sqrt{31.74 (31.74-21.7) (31.74-18.9) (31.74-22.87)} = 190.51$$

Ans: Area of quadrilateral ABCD = 190.51 + 48.90 = 239.4052.

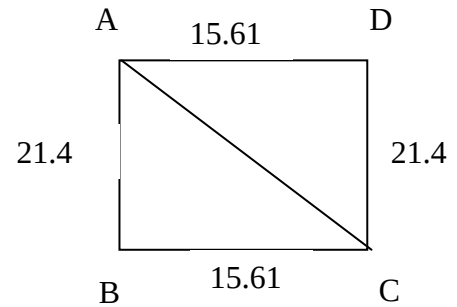
24. Two adjacent sides of a rectangle are 21.4m and 15.61m respectively. Find out the angle which a diagonal of the rectangle makes the side and length of the diagonal.

$$\tan \angle C = \frac{21.4}{15.61} = 53^{\circ}53'29.47''$$

$$\tan \angle A = \frac{15.61}{21.4} = 36^{\circ}6'30.53''$$

$$\frac{BC}{AC} = \sin \angle A$$

$$AC = \frac{BC}{\sin \angle A} = 26.49 \text{ yds.}$$



Ans: $\angle C = 53^{\circ}54'29.47''$, $\angle A = 36^{\circ}6'30''$ and $AC = 26.49$ yds.

25. ABC is a triangle with sides BC=36 cm, CA=25 cm, AB=29 cm. A point O lies in side the triangle and is distant 5 cm from BC and 10 cm from CA. Find is distance from AB.

From $\triangle ABC$

$$\begin{aligned} \cos \angle C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{36^2 + 25^2 - 29^2}{2 \times 36 \times 25} \\ &= 57^{\circ}7'48.37'' \end{aligned}$$

$$\angle DBC = 90^{\circ} + 57^{\circ}7'48.37'' - 180^{\circ} = 32^{\circ}52'11.6''$$

$$\angle ABD = 13^{\circ}31'13.76''$$

$$\frac{C}{\sin \angle C} = \frac{b}{\sin \angle B}$$

$$\sin \angle B = b \times \sin \angle C + c = 25 \times \sin 57^{\circ}7'48.37''$$

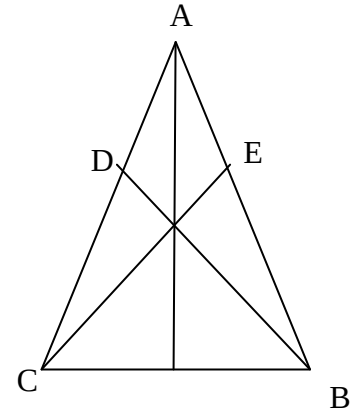
$$= 46^{\circ}23'25.39''$$

$$\text{In } \triangle CBD = \sin \angle c = \frac{P}{H} = \frac{DB}{CB}$$

$$DB = \sin 57^{\circ}7'48.37'' \times 36 = 30.24 \text{ cm}$$

$$\text{Now In } \triangle EOB = \sin \angle o = \frac{P}{H} = \frac{OB}{OE}$$

$$OE = \sin 13^{\circ}31'13.76'' \times 30.24 \text{ cm} = 4.73 \text{ cm}$$



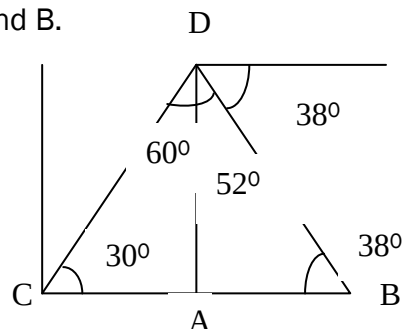
Ans: Dstn AB = 4.73 cm.

26. Two land marks A and B are observed by a man to be at the same instant in a line due east. After he has walked 4.5 km in a direction 30° north of east, A is observed to be due south while B is 38° south of east. Find the distance between A and B.

$$\frac{P}{B} = \tan \angle 38^\circ$$

Or $B = \frac{P}{\tan 38} = 2.8799 \text{ km}$

Ans: Dstn between A and B is 2.8799 km.



27. You were walking through a public road in the city of Djibouti. Suddenly you are struck by garbage thrown at your head by house wife from the multistoried residence beside the road. You could not see from which storey it was thrown but somehow measured that the garbage is striking at marking $60^\circ-23'$ angle with the horizontal. You measured the distance from the building to be 53.436 feet, and you know in feet makes a storey. Find out the storey from which it was thrown. By the way you are 6' tall.

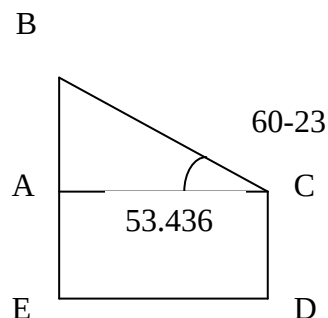
$$\tan \angle C = \frac{AB}{AC}, \text{ or, } AB = AC \times \tan \angle C$$

$$\text{or, } AB = 1.7278 \times 0.2453 = 1.9731.$$

$$\text{or, } AB = 1.9731 = 94.0'$$

$$AB + BE \div 10' = 94.0 + 6' \div 10'$$

$$= 100 \div 10 = 10'$$



Ans: It was thrown from the 10th storey.

28. A duck was sitting at a corner of a rectangular pond having the length of two adjacent sides to be 54 and 98 yards respectively. A jackal seeing the duck rustled to it but before the jackal could catch. The duck jumped in to the pond and started swimming diagonally at a speed of 1.5' per second. The jackal also started running along the bank at double the speed, prove mathematically whether the jackal will be able to catch the duck on the other side or not.

a. $\tan \angle BDC = \frac{BC}{BD}, \tan \angle BDC = 1.9912$

$$= 1.7412 \log$$

$$\angle BDC = 28^\circ 51' \text{ of } \tan \angle BDC$$

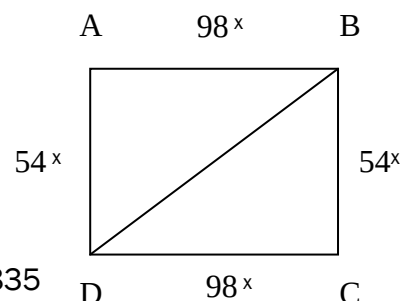
$$\text{Or, } \angle BDC = 28^\circ 51' - 20.38''$$

b. $\sin \angle BDC = \frac{BC}{BD}, \sin \angle BDC = 1.7324 - 9.6835$

$$= 2.0480 + \text{factor } 0.4771 = 2.5260$$

$$= 335.8' = 111.9^\circ \text{ or, } 111.89^\circ$$

c. $335.8' + 1.5' = 2.5260 - 0.1761 = 2.3499 = 225.9' \text{ sec time } 223.79'' \text{ time is required for duck.}$



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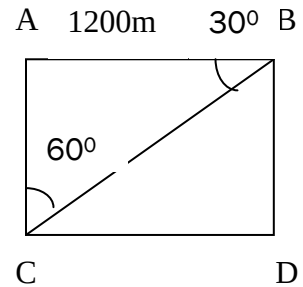
d. $54x + 98x = 152x \div 3' (1 \text{ sec} = 3') = 152 \times 3 \div 152 \text{ sec}$ time is required for Jackle.

Ans: Yes the Jackle will be catch the duck.

29. A mig 27 was flying at a speed of mach 2 when Gnr Lal miah saw it exactly above him. He immediately tried to angle the ac and finally succeeded in shooting at down after 1.5 secs. The angle of elevation measured from the AA Gun is 30° . What was the altitude of the ac (Speed of sound is 400 M/sec).

$$\begin{aligned}\tan \angle B &= \frac{AC}{AB}, \\ AC &= AB \times \tan \angle B \\ &= 1200 \text{ m} \times 30^\circ \\ &= 3.0792 + 9.7614 \\ &= 2.8406 = 692.8 \text{ m}\end{aligned}$$

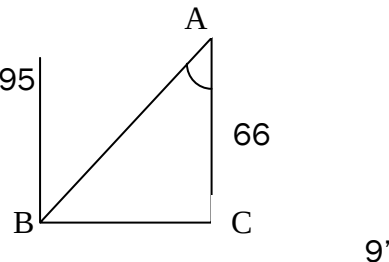
Ans: 692.8 m.



30. A stone was dropped from the tower of pisa at a constant velocity of 20' per sec. A man down was hit on the head by the stone while he had walked 3 secs at, a velocity of 1 yd per sec from the base of the tower in straight line. What was the towers angle of inclination with the vertical if the man was 6 feet tall. (The man started walking at the same instant the stone was dropped).

$$\begin{aligned}\tan \angle A &= \frac{BC}{AC} = \tan \angle A = 0.9542 - 1.8195 \\ \tan \angle A &= 9.1347\end{aligned}$$

Ans: $\angle A = 07^\circ - 46'$.

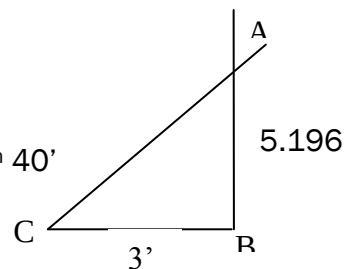


31. Sun set on 17 Dec 1421 was 1800 hrs and at 1300 hrs the sun was exactly vertical over Lt kudrat Ali some where in Sahara desert. After some time he planted his bty centre Rod which was 5.196 feet long and found the shadow to be only 3 feet long. What was the time at that instant ? (BC rod to be assumed exactly vertical).

$$\begin{aligned}\tan \angle C &= \frac{AB}{CB} = 0.7157 - 0.4771 = 0.2386 \\ \angle C &= 60^\circ\end{aligned}$$

$$1300 \text{ hrs} + 1^h 40' = 1440 \text{ hrs } 30' \quad \frac{05 \times 30}{90} = \frac{05}{3} = 1^h 40'$$

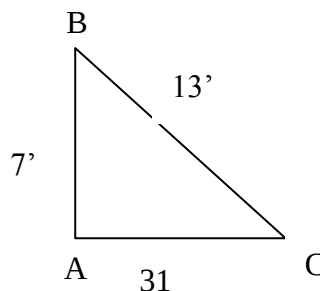
Ans: The time was 1440 hrs on that instant.



32. A pole of 20 feet broke at a height of 7 feet from the ground. The acute angle between the two parts of the pole will be (Write the correct Ans).

- $320^\circ 41' 22''$.
- II Radian.
- $57^\circ 48'$
- About 25 yards.

$$\begin{aligned}\cos \angle B &= \frac{7}{13}, \\ &= \log 7 - \log 13\end{aligned}$$



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$$= 0.8451 - 1.1139$$

$$= 9.7212, \quad \angle ABC = 57^{\circ} 25'$$

Ans: The acute angle between the two parts of the pole $57^{\circ} 25'$.

33. ABC is a triangle whose $\angle ACB = 90^{\circ}$, $\angle ABC = 38^{\circ} 53'$ Side BC = 201'. Find out side AB = AC.

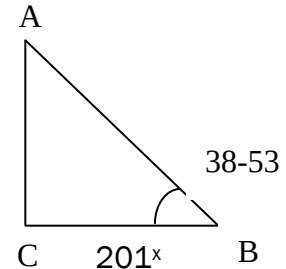
$$\frac{P}{B} = \tan \angle B$$

$$AC = 162.09'$$

$$\frac{P}{H} = \sin \angle B$$

$$AB = 258.21'$$

$$AC = 162.09'$$



Ans: $AB = 258.81'$ yds and $AC = 162.09'$.

34. ABC is a Rt angle Triangle, side AC is 609.9 yds and AB is 421.8 yds. Find out angle $\angle BAC$ and $\angle ACB$.

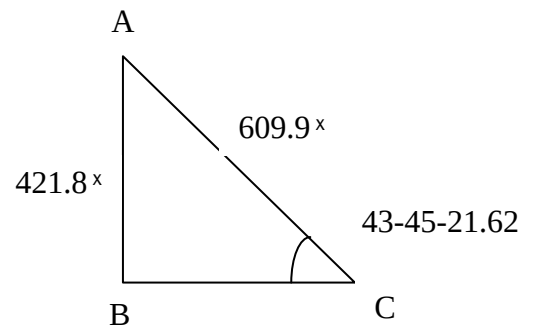
$$\frac{P}{H} = \sin \angle C$$

$$\sin \angle C = \frac{AB}{AC}$$

$$\angle C = 43^{\circ} 45' 21.62''$$

$$\frac{B}{H} = \cos \angle A$$

$$\cos \angle A = \frac{AB}{AC}$$



Ans: $\angle BAC = 46-14'-39.38''$ and $\angle ACB = 43-45-21'.62''$.

35. A pole and a palm tree are on the same horizontal plane. The ht of the tree is 120 feet. if a man sitting on the top of the tree measures angle of sight to the top and bottom of the pole as $10^{\circ} 30' 00''$ elevation and $15^{\circ} 40'$ depression respectively find out the ht of the pole.

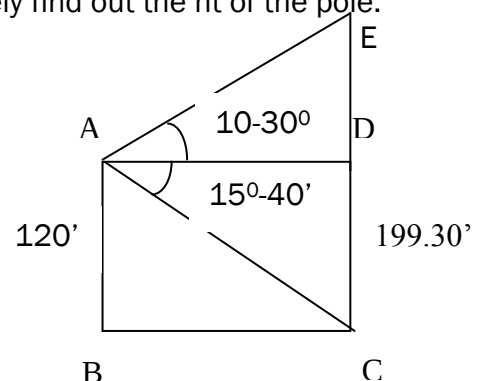
$$\text{From } \triangle ABC, \tan 15-40 = \frac{120}{BC} \quad \text{Or,} \quad \frac{120}{\tan 15-40}$$

$$BC = 427.87'$$

$$\text{Now In } \triangle AED, \tan 10-30 = \frac{DE}{427.87}$$

$$\text{Or, } DE = 427.87 \times \tan 10-30'$$

$$\text{Or, } DE = 79.30'$$

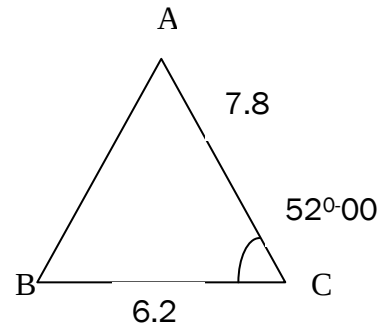


Ans: Ht of the pole is $(120+79.30') = 199.30$ ft.

36. Find the area of a triangle, when $a = 6.2$ m, $b = 7.8$ m and angle $\angle ACB = 52^\circ$.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times ab \times \sin Q \\ &= \frac{1}{2} \times 6.2 \times 7.8 \times \sin 52^\circ \\ &= 19.054 \text{ SQM} \end{aligned}$$

Ans: 19.054

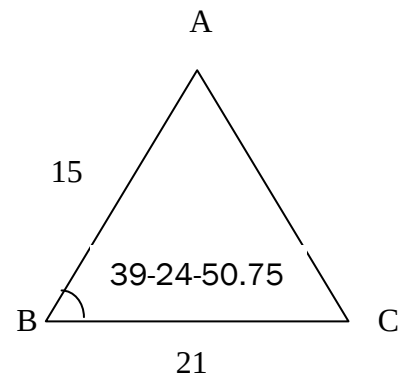


37. If the area of a triangle is 100m SQM and two of its sides are 21m and 15m find out the angle between these sides.

We know area of $\triangle ABC$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times ab \times \sin Q \\ \text{Or, } \sin Q &= \frac{100}{\frac{1}{2} \times 21 \times 15} \\ \angle B &= 39^\circ 24' 50.75'' \end{aligned}$$

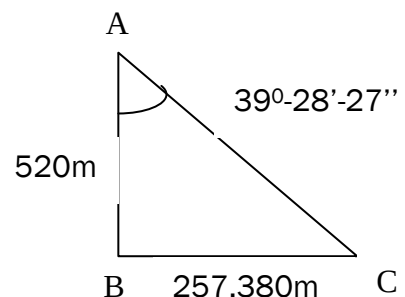
Ans: $\angle B = 39^\circ 24' 50.75''$.



38. ABC is a RT angle triangle. $AB = 520$ m $\angle BAC = 39^\circ 28' 27''$. $BC = 257.380$ m. Find out area of the triangle.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{ht} \\ &= \frac{1}{2} \times 257.380 \times 520 \\ &= 66918.8 \end{aligned}$$

Ans: 66918.8



Find out the value of $\log$ with the help of $\log$:

$$\begin{aligned} 39. \quad &0.364075 \times 0.0001 \\ &= \log 1.5612 + \log 4.0000 \\ &= \log 5.5612 \\ &= 0.00003641 \text{ Ans} \end{aligned}$$

$$\begin{aligned} 40. \quad &0.00019 \times 4637 \\ &= \log 4.2788 + \log 3.6662 \\ &= 6.2788 + 3.6662 \\ &= 9.9450 \\ &= 0.8810 \text{ Ans} \end{aligned}$$

41. $(0.0046 \times 1.47)^3$
 $= 3 \times \text{Log } (/3.6628 + 0.1673)$
 $= 3 \times 3.8301$
 $= 3 \times 7.8301$
 $= /3.4903$
 $= 3092.4309 \text{ Ans}$
42. $\text{Cos } 48^\circ - 27' \times 3986$
 $= \text{Log } 9.8217 + \text{Log } 3.6005)$
 $= 3.4222$
 $= 2643.6259 \text{ Ans}$
43. $\text{Sin } 40^\circ - 12' \times 0.1649$
 $= \text{Log } 9.8099 + \text{Log } /1.2172)$
 $= /1.8099 + \text{Log } /1.2172$
 $= /1.0271$
 $= 0.1064 \text{ Ans}$
44. 139×1640
 $= \text{Log } 2.1139 + \text{Log } 3.2148$
 $= \text{Log } 5.3287$
 $= 213157.1967 \text{ Ans}$
45. $.00306 \times 1728$
 $= \text{Log } /3.4857 + \text{Log } 1.2375$
 $= /4.7232$
 $= 0.0005287 \text{ Ans}$
46. 9999.9×777.2
 $= \text{Log } 4.0000 + \text{Log } 2.8905)$
 $= 6.8905$
 $= 7772000.001 \text{ Ans}$
47. $\text{Sin } 41^\circ - 20' \times .00169$
 $= \text{Log Sin } /1.8198 + \text{Log } /3.2279)$
 $= /3.0477$
 $= 0.001116 \text{ Ans}$
48. $.13 \times 5.0939$
 $= \text{Log Sin } /1.1139 + \text{Log } 0.7071)$
 $= /1.8210$
 $= 0.06621 \text{ Ans}$
49. $\text{Cos } 35^\circ - 19' \div 4690$
 $= \text{Log } /1.9117 - 3.6712)$
 $= /4.2405$
 $= 0.000173980 \text{ Ans}$

50. $\sin 45^\circ - 12 \div .1649$
 $= \log / 1.8099 - \log / 1.2172$
 $= 0.5927$
 $= 3.9144 \text{ Ans}$
51. $(25.23)^2 \div .0853$
 $= \log (1.4019 \times 2 \div 3) - / 2.9309$
 $= \log (2.8038 \div 3) - / 2.9309$
 $= \log (0.9346 - / 2.9309)$
 $= 2.0037$
 $= 100.8556 \text{ Ans}$
52. $5.7 \div .0010$
 $= \log 0.7559 - / (3.0000)$
 $= 3.7559$
 $= 5700.33 \text{ Ans}$
53. $2742 \div 0001$
 $= \log 3.4381 - \log 0.0000$
 $= 3.4381$
 $= 2742.20 \text{ Ans}$
54. $1560 \div .0002$
 $= \log 3.1931 - \log / 4.3010$
 $= 6.8921$
 $= 7800096.937 \text{ Ans}$
55. $(.7391)^3 \div (.08832)$
 $= \log / 1.8687 \times 3 - \log / 2.9461$
 $= / 1.6061 - \log / 2.9461$
 $= 0.6600$
 $= 4.5709 \text{ Ans}$
56. $9.259 \div \sin 4885.7 / m$
 $= \log 0.9666 \div \sin 274-49-14.25''$
 $= \log 0.9666 - / 1.9665$
 $= 0.9681$
 $= 9.2926 \text{ Ans}$
57. $.004615 \div .00049$
 $= \log / 3.6642 - \log / 4.6902$
 $= 0.9740$
 $= 9.4189 \text{ Ans}$
58. $(.51)^2 \div (.309)^2$

$$\begin{aligned}
 &= \text{Log } /1.7076 \times 2 \div \text{Log } /1.4900 \times 2 \\
 &= \text{Log } /1.4152 - /2.9800 \\
 &= 0.4352 \\
 &= 2.7240 \text{ Ans}
 \end{aligned}$$

59. ABC is a triangle, whose $\angle A = 45-57-00$ AND $\angle B = 53-59-00$. Side AC is 443.0 yds. Find out side BC and AB.

$$\angle ABC = 180 - (48-57-00 + 53-59-00)$$

$$\angle C = 77-4-00$$

$$a = \frac{b \times \sin \angle A}{\sin \angle B}$$

$$a = \frac{443.3 \times \sin 48-57-00}{\sin 53-59-00}$$

$$\therefore BC = 413.32^x$$

$$\frac{c}{\sin \angle C} = \frac{a}{\sin \angle A}$$

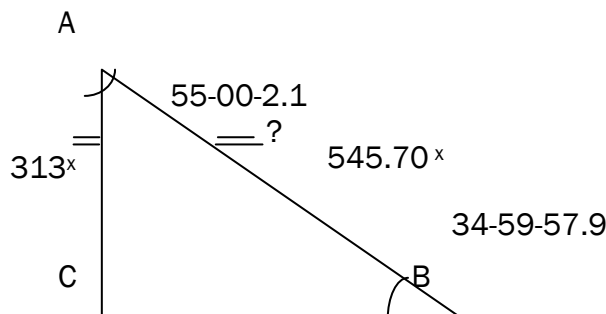
$$C \times \sin A = a \times \sin C$$

$$C = \frac{a \times \sin \angle C}{\sin \angle A}$$

$$C = \frac{413.32 \times \sin 77-04-00}{\sin 48-57-00}$$

$$\therefore AB = 534.17^x \text{ Ans}$$

60. ABC of a TRIANGLE, WHOSE $\angle ACB$ is 90° side AC is 313 yds $\angle BAC = 55-00-2.1$. Find out side BC and $\angle ABC$



$$\frac{AC}{AB} = \cos \angle A$$

$$AB \times \cos \angle A = AC$$

$$AB = \frac{AC}{\cos \angle A}$$

$$AB = \frac{313}{\cos 53-59-00}$$

$$AB = 545.70^x$$

For $\triangle ACB$

$$AB^2 = AC^2 + BC^2$$

$$\text{Or, } BC^2 = AC^2 - AB^2$$

$$\text{Or, } BC^2 = 545.70^2 - 313^2$$

$$\text{Or, } BC = \sqrt{545.70^2 - 313^2}$$

$$\therefore BC = 447.02 \text{ yds}$$

$$\frac{AC}{BC} = \tan \angle B$$

$$\text{Or, } \tan B = \frac{AC}{BC}$$

$$= \frac{313}{447.02}$$

$$\angle B = 34-59-57.9$$

$$\text{Ans : } BC = 447.02$$

$$\angle B = 34-59-57.9$$

$$61. \frac{\sin 45^0-30'-00'' \div \sqrt{(921.47)^5 \times 887.902 \div 07.35}}{(0.0029)^{3/4} \times \cos 75^0-11'-25'' \div 519.12 \div (51)^4}$$

$$\text{Ans: } 3675269.564$$

$$62. \frac{\sin 47^0-29'-57'' \times (0.00309)^{4/5}}{\tan 55^0-33'-03'' \times \cos 49^0-39'-41''}$$

$$\text{Ans: } 0.007669949$$

$$63. (0.0088)^{3/4} \div \sqrt{(0.0562)^{2/3}}$$

$$\text{Ans: } 0.75009551$$

$$64. \sqrt{(61.27)^3 \times (52.65)^2}$$

$$\text{Ans: } 25250.51243$$

$$65. (0.00132)^{3/2} \times (87.99)^2 \times (51.61)^{4/3}$$

$$\text{Ans: } 8.446685785$$

$$66. \frac{\sqrt{(15.92)^{1/7} \times (0.0182)^{1/3}}}{\sqrt{0.00526 \times (196)^5}}$$

Ans: 0.000016023

67.
$$\frac{\tan 59-12-10 \times \sin 47-37-10- (0.0792)^4}{(13.921)^{3/2} \times (57.49)^{3/2}}$$

Ans: 0.00054735.

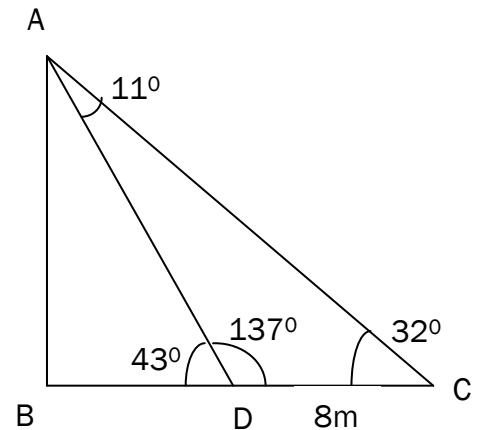
68.
$$\frac{\sqrt{(5621)^2 \div (41.56)^{1/2} \times (0.004126) \div 207^3}}{\sqrt{5 (5671)^3 \times 461 \div (561)^2}}$$

Ans: 10201.90788

69. A man observes that the angle of elevation of bldg is 32° . He walks 8m in a direct line towards the bldg and the finds that the angle of elevation 43° . What is the ht of the bldg ?

$$\begin{aligned}\angle ADB &= 180-43 = 137 \\ \angle BAD &= 180- (32+137) = 11^\circ \\ \text{ABD } \frac{BD}{\sin \angle A} &= \frac{AD}{\sin \angle B} \\ AD \times \sin \angle A &= BD \times \sin \angle B \\ AD &= \frac{BD \times \sin \angle B}{\sin \angle A} \\ &= \frac{8 \times \sin 32}{\sin 11} = 22.217 \text{ M}\end{aligned}$$

$$\begin{aligned}\triangle ADC, \sin D &= \frac{AC}{AD} \\ AC &= \sin 43 \times 22.217 = 15.151 \text{ m}\end{aligned}$$



Ans: Ht of the building is 15.151 m.

70. Co-ords of bc E 45750.00 N 78280.00 Ht 305`, Bty Cen to tgt Bg 45-30-00` Dstn 2550 Yds A/S 00-25-00, Find out the Co-ords tgt by math table.

Log Dstn	= 3.4065	Log Dstn	= 3.4065	Log Dstn	= 3.4065
(+)Log sin bg	= 9.8532	(+) Log cos bg	= 9.8457	(+) Log tan A/S	= 7.8617
				(+) Log factor	= .4771

Log d E	= 3.2597	Log d N	= 3.2522	Log d H	= 1.7453
d E	= 1819	d N	= 1787	d H	= 55.63
Co-ords of BC E 45750.00		N 78280.00		Ht 305`	
+ 1819		+ 1787		- 55.63	

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Co-ords of Tgt E 47568.00

N 80067.00

Ht 249.37

71. Co-ords of bc E 00721.00 N 89527.00 Ht 1325`, Bty Cen to tgt Bg 335-00-00` Dstn 5225 Yds A/S 02-30-00, Find out the Co-ords tgt by math table.

Log Dstn = 3.7181	Log Dstn = 3.7181	Log Dstn = 3.7181
(+)Log sin bg = 9.6259	(+) Log cos bg = 9.9573	(+) Log tan A/S = 8.6401
		(+) Log factor = .4771

Log d E = 3.3440	Log d N = 3.6754	Log d H = 2.8353
d E = 2208	d N = 4735	d H = 684.30

Co-ords of BC E 00721.00	N 89527.00	Ht 1325`
- 2208	+ 4735.00	- 684.30

Co-ords of Tgt E 98513.00	N 94262.00	Ht 640.07
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72. Co-ords of P bty E 39469.20 N 27592.80 Ht 143.20`, P Bty to tgt Bg 210-27-13` Dstn 2130 Yds A/S 00-13-00, Find out the Co-ords tgt by math table.

Log Dstn = 3.3284	Log Dstn = 3.3284	Log Dstn = 3.3284
(+)Log sin bg = 9.7048	(+) Log cos bg = 9.9355	(+) Log tan A/S = 7.5777
		(+) Log factor = .4771

Log d E = 3.0332	Log d N = 3.2639	Log d H = 1.3832
d E = 1079	d N = 1836	d H = 24.17

Co-ords of BC E 39469.20	N 27592.80	Ht 143.20`
- 1079	- 1836.00	- 24.17

Co-ords of Tgt E 38390.00	N 25756.00	Ht 119.03
---------------------------	------------	-----------

73. Co-ords of BC E 03929.60 N 53996.51 Ht 16901.10`, CO-ORDS tgt 04319.73 N 54129.33 Ht 1530.00`, Find Out bg dstn and A/S?

Co-ords of BC E 03929.60	N 53996.51	Ht 16901.10`
CO-ORDS tgt E 04319.73	N 54129.33	Ht 1530.00`
d E 390.13	(+)d N 132.82	d HT (-) 160.1`
LOG d E 2.5912		
(-) LOG d N 2.1232		
LOG TAN Bg 0.4680,	Bg = 71-12-00	

Log d E = 2.5912	Log d N = 2.1232	Log LOG Dht = 2.2044
(-)Log sin bg = 9.9762	(-) Log cos bg = 9.5082	(-) factor = .4771

$$\begin{aligned} \text{Log dstn} &= 2.6150 & \text{Log dstn} &= 2.6150 & \text{Log d Ht} &= 1.7273 \\ \text{dstn} &= 412.1 \text{ yds} & & & \text{(-) LOG dstn} &= 2.6150 \\ & & & & \text{LOG TAN A/S} &= 9.1123 \\ & & & & \text{A/S} &= 7=23-00 \end{aligned}$$

Co-ords of BC	E 76930.10	N 48191.10	Ht 1002.1`
CO-ORDS tgt	E 76001.90	N 48090.80	Ht 1103.7`
	d E 928.20	(-)d N 100.30	d HT (+) 101.60`

Log d E	= 2.9676	Log d N	= 2.0013	Log LOG Dht	= 2.0068
(-)Log sin bg	= 9.9975	(-) Log cos bg	= 9.0311	(-) factor	=0.4771

Ans: Bg = 263-50-00
Dstrn = 933.4 yds
A/S = 02-05-00

Co-ords of BC	E 03929.51	N 53996.51	Ht 11690
CO-ORDS tgt	E 03729.10	N 52829.53	Ht 1790.0
	(-) d E 200.41	(-)d N 1166.98	d HT (+) 100

Log d E	= 2.3018	L0g d N	= 3.0667	Log LOG Dht	= 2.0000
(-)Log sin bg	= 9.2288	(-) Log cos bg	= 9.9937	(-) factor	=0.4771

$$\begin{array}{llll} \text{Log dstn} & = 3.0730 & \text{Log dstn} & = 3.0730 & \text{Log d Ht} & = 1.5229 \\ \text{dstn} & = 933.4 \text{ yds (Anti Log)} & & & \text{(-) LOG dstn} & = 3.0730 \\ & & & & \text{LOG TAN A/S} & = 8.4499 \end{array}$$

A/S= 01-37-00

Ans: Bg =189-45-00
Dstn = 1183.0 yds
A/S = 01-37-00

76. Co-ord of bty cen E: 19900 N: 39300 HT: 615'. Co-ord of tgt E: 15850.00 N:35650.00
Ht : 515'. Find out the bg, dstn and A/S of tgt.

S POL 35650 (-) 39300 , 15850 (-) 19900 EXE = 5452.06

Y EXE S DEG ANS EXE = 227-58-43

515 (-) 615 EXE (/) 3 (/) 5452.06 EXE S TAN ANS EXE S DEG ANS EXE =
00-21-1.07

Ans: Bg 227-58'-25.43'', Dstn 5452.06 and A/S 0-21'-1.07''.

77. Co-ords of P bty cen E 49520 N 99370 Ht 750'
Co-ords of tgt E 53170 N 02550 Ht 890'
Find bg, dist and A/S to the tgt by cal.

S POL 02550 (-) 99370 , 53170 (-) 49520 EXE = 6395.70

Y EXE S DEG ANS EXE = 219-17-21.86

890 (-) 750 EXE (/) 3 (/) 6395.70 EXE S TAN ANS EXE S DEG ANS EXE =
(-) 00-17-55.01

Ans: Bg = 219-17-21.86, Dist = 6395.70 and A/S = (-) 17-55.01

78. Co-ords of bty cen E 19500 N 40600 Ht 615'
Co-ords of tgt E 15450 N 35650 Ht 515'
Find bg, dist and A/S to the tgt by math table.

<u>S</u>		<u>POL</u>		<u>35650</u>	(-)	<u>40600</u>	(,)	<u>15450</u>	(-)	<u>19500</u>	<u>EXE</u>	=	6395.70
<u>Y</u>		<u>EXE</u>		<u>S</u>		<u>DEG</u>		<u>ANS</u>		<u>EXE</u>	=		219-17-21.86
<u>515</u>	(-)	<u>615</u>		<u>EXE</u>	(÷)	<u>3</u>	(÷)	<u>6395.70</u>		<u>EXE</u>	S		Tan
<u>ANS</u>		<u>EXE</u>		<u>S</u>		<u>DEG</u>		<u>ANS</u>		<u>EXE</u>	=	(-)17-55-01	

Ans: Dist = 4840.960, Bg = 48-56-11.58 and A/S = (+) 00-33-8.32

RESTRICTED

79. Co-ords of P bty cen E 49520 N 99370 Ht 750'
Co-ords of tgt E 53170 N 02550 Ht 890'
Find bg, dist and A/S to the tgt by cal.

S POL 02550 (-) 99370, 53170 (-) 49520 EXE=
Y EXE S DEG ANS EXE

890 (-) 750 EXE (/) 3 (/) DSTN EXE S TAN ANS EXE S DEG ANS EXE =

80. Co-ords of sta A is E 03450.00 N 67650.00 Ht 50.00' ft. bg and dstn and A/S from "A" to "B" are 245° and 5300 yds and (+) 00°-54'-56". Find out the loc of the sta "B".

<u>S</u>	<u>REC</u>	<u>5300</u>	(.)	<u>245°</u>	<u>EXE</u>	(+)	<u>67650</u>	<u>EXE</u>
<u>65410.12</u>	<u>Y</u>	<u>EXE</u>	(+)	<u>03450</u>	<u>EXE</u>		<u>98646.57</u>	
<u>5300</u>	<u>x</u>	<u>3</u>	<u>X</u>	<u>Tan 00°-54'-56"</u>	<u>EXE</u>	(+)	<u>50</u>	
<u>EXE</u>	<u>=</u>							

Ans: Sta B E 98646.57, N 65410.12

81. Co-ords of bty cen E 52870 N 40345 Ht 27'
Co-ords of tgt E 57870 N 42345 Ht 35'
Find out bg, dist and A/S by cal.

<u>S</u>	<u>POL</u>	<u>42345</u>	(-)	<u>40345</u>	(.)	<u>57870</u>	(-)	<u>52870</u>	<u>EXE</u>	= 5385.16
<u>Y</u>	<u>EXE</u>	<u>S</u>		<u>DEG</u>	<u>ANS</u>	<u>EXE</u>			=	68-11-54.93
<u>35</u>	(-)	<u>27</u>		<u>EXE</u>	(÷)	<u>3</u>	(÷)	<u>5385.16</u>	<u>EXE</u>	<u>S</u>
<u>ANS</u>	<u>EXE</u>	<u>S</u>		<u>DEG</u>	<u>ANS</u>	<u>EXE</u>			=	00-02-33.21

Ans: Bg = 68-11-54.93, Dist = 5385.16 and A/S = 00-02-33.21

82. Co-ords of P/Bty E 98356.30 M 74125.60 HT 150' and Co-ords of tgt E 99806.50 N 73256.70 HT 75'. Find out Bg, Dist and A/S from P/Bty to tgt.

<u>S</u>	<u>POL</u>	<u>73256.7</u>	(-)	<u>74125.6</u>	(.)	<u>99806.5</u>	(-)	<u>98356.3</u>	<u>EXE</u>	= 1190.58
		<u>0</u>		<u>0</u>		<u>0</u>		<u>0</u>		
<u>Y</u>	<u>EXE</u>	<u>S</u>		<u>DEG</u>	<u>ANS</u>	<u>EXE</u>			=	120-55-00
<u>75</u>	(-)	<u>150</u>		<u>EXE</u>	(÷)	<u>3</u>	(÷)	<u>1190.58</u>	<u>EXE</u>	<u>S</u>
<u>ANS</u>	<u>EXE</u>	<u>S</u>		<u>DEG</u>	<u>ANS</u>	<u>EXE</u>			=	00-50-49

Ans: Bg = 120-55'-00", Dist = 1190.58 and A/S = 00-50'-49".

RESTRICTED

83. Tgt co-ord E 41282.99 N 72100 ht 80'. Bty co-ord E 42282.99 N 71100.00 ht 89'. Find out Bg, dist and A/S to tgt from bty.

$$\begin{array}{rcllcllcllcll} \underline{S} & & \underline{POL} & & \underline{71100.0} & (-) & \underline{72100} & (,) & \underline{42282.9} & (-) & \underline{41282.9} & \underline{EXE} & =1414 \\ & & & & \underline{0} & & & & \underline{9} & & \underline{9} & & \\ \underline{Y} & & \underline{EXE} & & \underline{S} & & \underline{DEG} & & \underline{ANS} & & \underline{EXE} & = & 135-00-00 \\ \underline{89} & (-) & \underline{80} & & \underline{EXE} & & & & \underline{ANS} & & \underline{EXE} & \underline{S} & \underline{Tan} \\ \underline{ANS} & & \underline{EXE} & & \underline{S} & & \underline{DEG} & & \underline{ANS} & & \underline{EXE} & = & 00-07-00 \end{array}$$

Ans: Bg = 135-00'-00'', Dist = 1414 and A/S = 00-07'-00''.

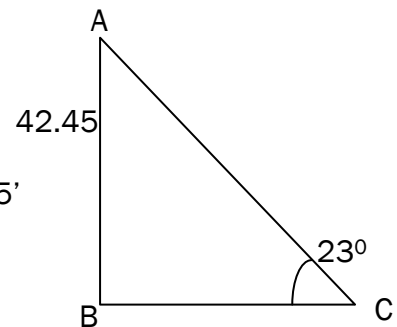
84. A man standing 100 feet away from a wall he measured the angle of sight to the wall 23°. Find out the ht of the wall.

In $\triangle ABC$

$$\frac{AB}{BC} = \tan \angle C$$

$$\text{Or, } AB = BC \times \tan \angle C = 100 \times \tan 23^\circ = 42.45'$$

Ans: 42.45'



85. The angle of ele of top a mountain from the bottom of tower 180m high angle is 26°. 25'. From the top of the tower the angle of ele is 25° -18'. Find the ht of the mountain.

From $\triangle ACD$, $\angle DCA = 90 + 25 - 18 = 115^\circ - 18'$

$$\angle CAD = 180^\circ - (115^\circ - 18' + 64^\circ - 25') = 00^\circ - 17'$$

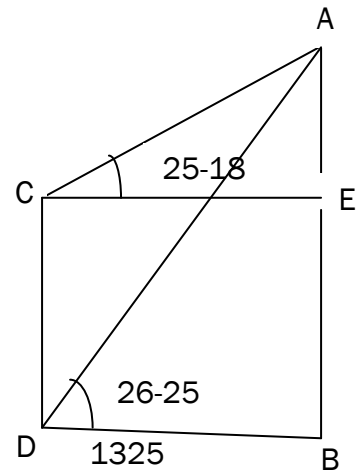
$$\angle ADC = 90^\circ - (26^\circ - 25') = 63^\circ - 35'$$

$$\frac{180}{\sin 00^\circ - 17'} = \frac{AD}{\sin 115^\circ - 18'}$$

$$AD = 73249.411$$

$$\sin 26^\circ - 25' = \frac{AB}{AD}$$

Ans: Ht of mountain is 32588.349



86. To tower A and B are apart by 1325m. From tower A you measured angle of depression to tower B is $7^{\circ}21'32''$. The ht of the tower B is 170m. Find the ht of the tower A.

AS $AE \parallel PB$

$$\angle EAP = \angle ABP = 7-21-32'$$

Again, $PD \parallel BC$

$$DC \parallel PB$$

$$PB = DC = 1325 \text{ m}$$

$$\text{In } \triangle APB, \tan B = \frac{AP}{PB}$$

$$\text{Or, } \tan 7-21-32 \times 1325 = AP$$

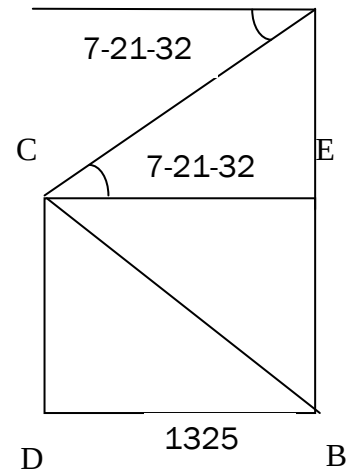
$$\text{Or, } AP = 171.120$$

$$AD = AP + PD$$

$$= 171.120 + 170$$

$$= 341.120 \text{ M}$$

Ans: 341.120 m.



87. The length of hypotenuse of right angle triangle is 250 metre. The hypotenuse and base makes a angle of 37° with ground. Find the height of perpendicular and length of the base of that right angle triangle.

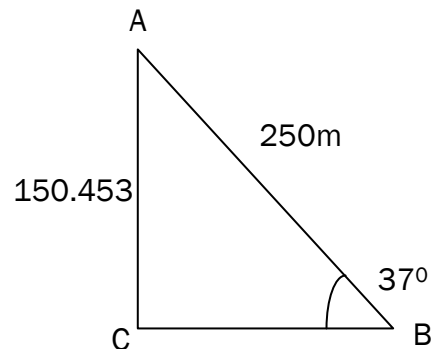
$$\cos 37 = \frac{AC}{250}$$

$$BC = 199.658$$

$$\sin 37^{\circ} = \frac{AC}{250}$$

$$AC = 150.453$$

Ans: $AC = 150.453$



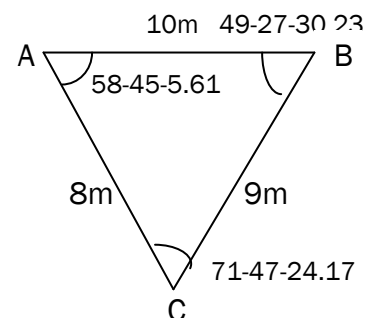
88. A wt was hung from a horizontal beam by two chains 8m and 9m long respectively. The end of the chain is being fastened to the beam at pt 10m apart. Determine ele angle. Which the chain makes with the beam.

$$\text{We know } \cos \angle A = \frac{10^2 + 8^2 - 9^2}{2 \times 10 \times 8}$$

$$\angle A = 58-45-5.61$$

$$\text{Again, } \cos \angle B = \frac{10^2 + 9^2 - 8^2}{2 \times 10 \times 9}$$

$$\angle B = 49-27-30.23$$



$$\begin{aligned}\angle C &= 180 - (\angle A + \angle B) \\ &= 71-47-24.17\end{aligned}$$

Ans: $71^{\circ}-47'-24.17''$

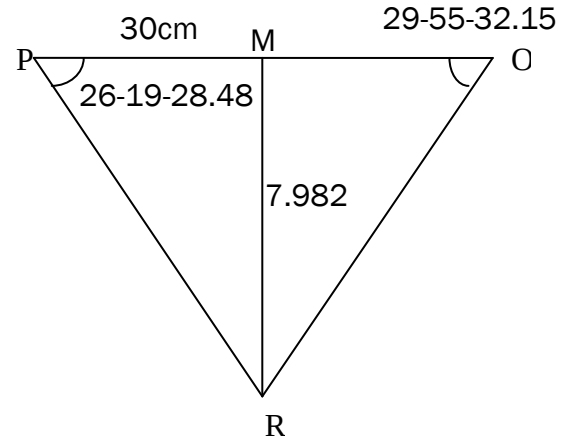
89. Two hooks 'p' and 'o' on a horizontal beam 30 cm apart. From 'p' and 'o' slings PR 18 cm and 16 cm long respectively support a wt at R. Find the dist of R from the beam and the angle which PR and OR make with beam.

$$\begin{aligned}\text{For } \triangle PRO, \cos \angle P &= \frac{30^2 + 18^2 - 16^2}{2 \times 30 \times 18} \\ P &= 26-19-28.48\end{aligned}$$

$$\begin{aligned}\text{For } \triangle PRM, \frac{RM}{PR} \sin 26-19-28.48 \\ RM &= 7.982 \text{ cm}\end{aligned}$$

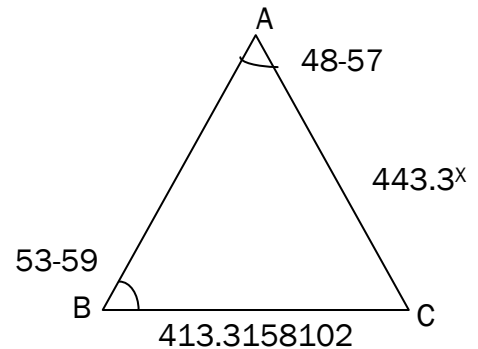
$$\begin{aligned}\text{For } \triangle MRO, \sin \angle O &= \frac{RM}{PR} = \frac{7.982}{16} \\ O &= 29-55'-32.15''\end{aligned}$$

Ans: $29^{\circ}-55'-32.15''$



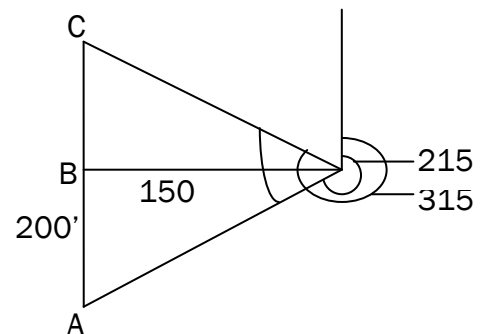
90. ABC is a triangle $\angle A$ is $48-57'$ and $\angle B$ is $53-59'$, side AC is 443.3 yds, Find out side AB and AC.

$$\begin{aligned}\frac{AB}{\sin \angle C} &= \frac{AC}{\sin \angle B} \\ AB &= \frac{443.3 \times \sin 77-44}{\sin 53-59} = 534.1610123 \\ \frac{BC}{\sin \angle A} &= \frac{AC}{\sin \angle B} \\ BC &= \frac{443.3 \times \sin 48-57}{\sin 53-59} = 413.3158102\end{aligned}$$



91. B and C are two pts on the opposite bank of the river Tista. P is a pt due to east of pt B along the same River bank and 150 ft away. A is a due south of pt B and 200 ft away making ABC a straight line. Bg from P to A is $215^{\circ}-30'-00''$ and P to C is $315^{\circ}-30'-00''$. Find out the width of the River.

$$\begin{aligned}\angle APC &= (315-30) - (215-30) = 100^{\circ} \\ \triangle ABP &= \tan \angle APB = \frac{200}{150} = 53-7-48.37 \\ \triangle BPC &= \angle BPC = 100 - (53-7+48.37) \\ &= 46-52-11.63 \\ \triangle PBC &= \tan 46-52-11.63 = \frac{BC}{150} \\ BC &= 160.124\end{aligned}$$



Ans: Width of the River is 160.124

92. A man running on a circular path by 10 miles per hour. After running 36 sec he subtends an angle of 56° on the centre. Find out the diameter of the circle .

In 3600 sec run 10 miles

$$\begin{array}{lcl} 1 & \text{''} & \frac{10}{3600} \text{''} \\ 36 & \text{''} & \frac{10 \times 36}{3600} = .1 \text{ miles} \end{array}$$

We know

$$56 = \frac{2\pi}{360} \times 56 \text{ Radians} = Q$$

Again, arc of a circle $S = r \times Q$

$$\begin{aligned} \frac{S}{Q} &= \frac{176 \times 360}{2\pi \times 56} \\ r &= 180.72^x = 1080.43 \end{aligned}$$

93. The large side of a triangle is 3.6 km long. Two of the angle are $37^\circ-15'$ and $48^\circ-24'$. Find out the greatest side.

$$\angle BAC = 180 - (37-15+48-24)$$

$$\angle A = 180 - (85-39-00)$$

$$\angle A = 94-21-00$$

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$$

Or, $a \times \sin \angle B = b \times \sin \angle A$

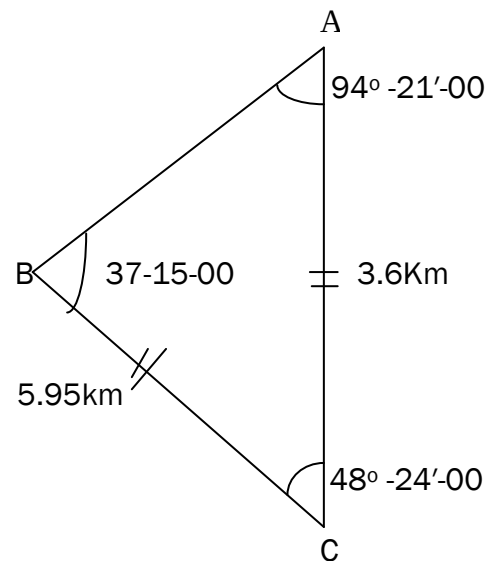
Or, $a = \frac{b \times \sin \angle A}{\sin \angle B}$

Or, $a = \frac{3.6 \times \sin 94-21-00}{\sin 37-15-00}$

$$S = 55.4$$

$$BC = 5.93 \text{ km}$$

Ans: The greatest side is 5.93 km.



94. ABC is a triangle, whose $a = 27.2\text{cm}$, $b = 32.8\text{cm}$ and $c = 50.8\text{cm}$. Find the area of triangle.

$$S = \frac{a+b+c}{2}$$

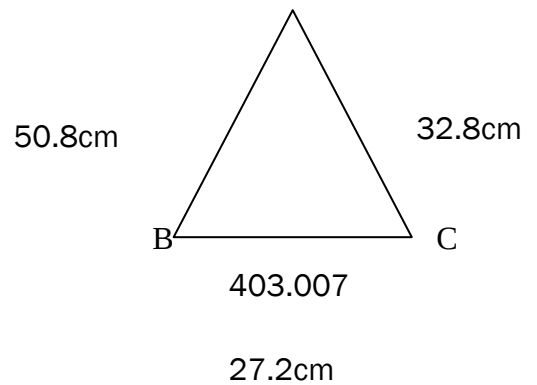
$$S = \frac{27.2+32.8+50.8}{2}$$

$$S = 55.4$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= 403.0070 \text{ cm.}$$

Ans: Area of triangle is 403.0070 cm.



95. Find the area of triangle when ABC when $a = 88.2\text{cm}$, $c = 70.5\text{cm}$ $\angle ABC = 130-20'-00''$.

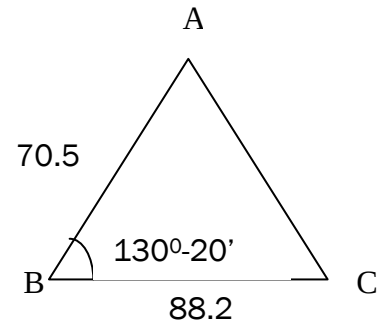
$$\frac{1}{2} \times ab \times \sin Q$$

$$= \frac{1}{2} \times a \times c \times \sin \angle B$$

$$= \frac{1}{2} \times 88.2 \times 70.5 \times \sin 130-20$$

$$\text{Area} = 2370.003674 \text{ cm.}$$

Ans: Area of triangle is 2370.003674 cm.



96. ABC is a Rt angle triangle whose $\angle ACB = 90^{\circ}$, $\angle ABC = 40^{\circ}-40'$ side AB = 530m, Find out side AC, BC and $\angle BAC$.

$$\frac{P}{H} = \sin \angle B$$

Or, $b = 530 \times \sin 40-40$

$$AC = 345.378$$

$$\frac{P}{B} = \tan \angle B$$

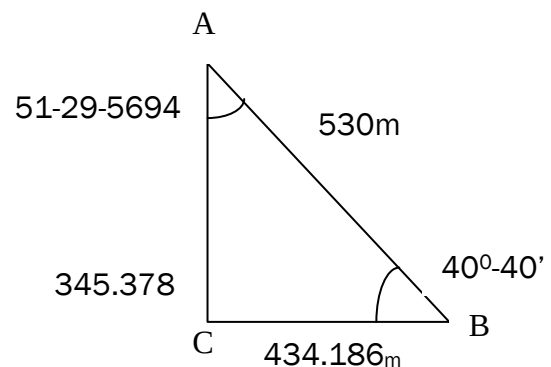
Or, $b = \frac{373.02}{\tan 40-40}$

$$BC = 434.186 \text{ M.}$$

$$\frac{P}{B} = \tan \angle A = \frac{434.186}{345.378}$$

$$\angle A = 51-29-56.94$$

Ans: $AC = 345.378$, $BC = 434.186\text{m}$ and $BAC = 51-29-56.94$.



97. ABC is a Rt angle triangle whose $\angle ACB = 90^\circ$, $\angle ABC = 39-35$ side AC = 162.09x, Find out side AB and AC.

$$\frac{P}{B} = \tan \angle B$$

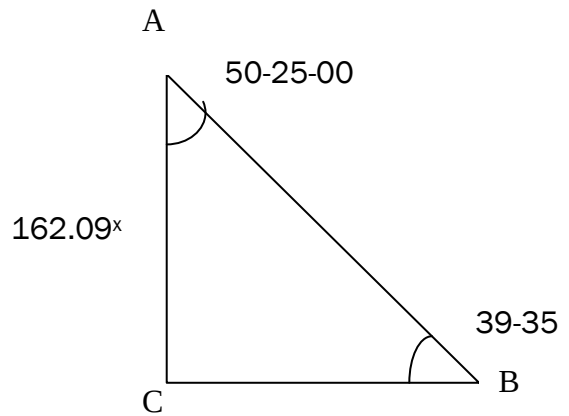
Or, $b = \frac{p}{\tan \angle B} = \frac{162.09}{\tan 39-35}$

AC = 196.049 .

$$\frac{P}{B} = \tan \angle A$$

Or, $\tan \angle A = \frac{P}{B} = \frac{196.049}{162.09}$,

$\angle A = 50-25-00$



98. ABC is a triangle $\angle B = 58-29-00$, $\angle C = 61-27-00$, side BC is 310 yds, Find out side AB and AC ?

$$\angle B = 180 - (58-29-00) + (61-27-00) = 60^\circ-04'-00$$

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$$

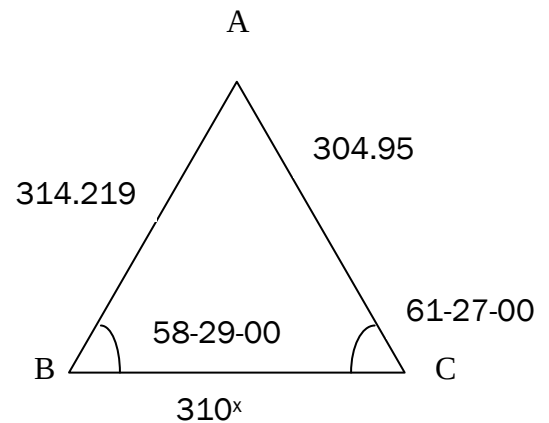
Or, $b = \frac{a \sin \angle B}{\sin \angle A}$

AC = 304.95

$$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

Or, $c = \frac{b \sin \angle C}{\sin \angle B}$

AB = 314.219



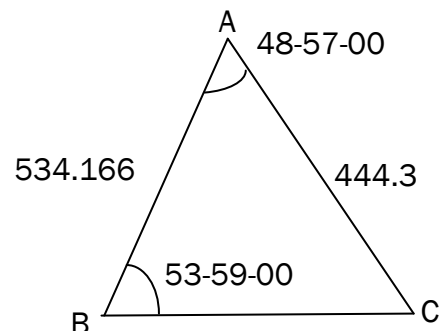
Ans: AB = 314.219, AC = 304.95

99. ABC is a triangle whose angle $\angle A$ is $48-57-00''$, $\angle B = 53-59-00$, side AC is 444.3 yds, Find out side AB and BC ?

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B}$$

Or, $\frac{BC}{\sin \angle A} = \frac{AC}{\sin \angle B}$

$= BC \times \sin \angle B = AC \times \sin \angle A$



$$BC = \frac{AC \times \sin \angle A}{\sin \angle B}$$

$$BC = \frac{443.3 \times \sin 48^\circ - 57' - 00''}{\sin 53^\circ - 59' - 00''}$$

$$BC = 413.32^x$$

$$\frac{a}{\sin \angle A} = \frac{c}{\sin \angle C}$$

$$a \times \sin \angle C = c \times \sin \angle A$$

$$\text{Or, } c = \frac{b \times \sin \angle C}{\sin \angle A} = \frac{413.32 \times \sin 77^\circ - 04' - 00''}{\sin 48^\circ - 57' - 00''}$$

$$AB = 534.166 \text{ yds}$$

Ans: $AB = 534.166 \text{ yds}$ and $BC = 413.32^x$

100. ABC is a triangle $\angle ABC$ is 90° , $\angle ACB$ is 42° . Side AB is 30m and Side AC is 45m. Find out $\angle BAC$ and side BC.

$$\frac{P}{b} = \tan \angle C$$

$$\text{Or, } b \times \tan \angle C = P$$

$$\text{Or, } b = \frac{P}{\tan \angle C}$$

$$\text{Or, } b = \frac{30}{\tan 42^\circ}$$

$$BC = 33.32 \text{ M.}$$

$$\frac{P}{b} = \tan \angle A$$

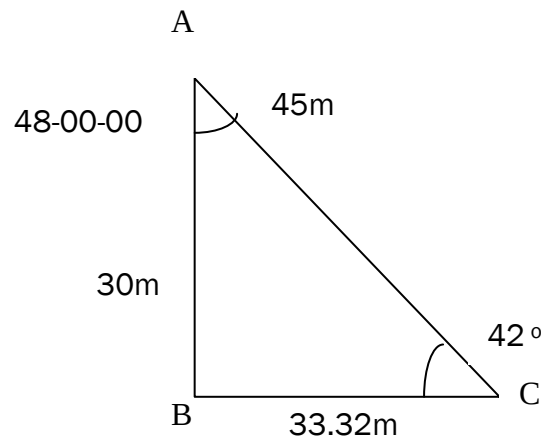
$$\text{Or, } b \times \tan \angle A = P$$

$$\text{Or, } b \times \tan \angle A = \frac{b}{P}$$

$$\text{Or, } \angle BAC = \frac{33.32}{30}$$

$$\angle BAC = 48^\circ - 00' - 00''$$

Ans: $BC = 33.32 \text{ M}$, $\angle BAC = 48^\circ - 00' - 00''$.



101. Two men stand on opposite sides of a church steeple and in the same straight line with it. They are 1500 metre apart. From one the angle of elevation to the top of church steeple is $15^\circ - 20' - 00''$ and from other it is $28^\circ - 40' - 00''$. Find out the height of the church steeple.

Ans: Ht is 273.917m.

102. A pole and a palm tree are on the same horizontal plane. The height of the tree is 120 feet. If a man sitting on the top of the tree measures angle of sight to the top and bottom of the pole as $10^{\circ}-30'-00''$ elevation and $15^{\circ}-40'-00''$ depression respectively. Find out the height of the pole.

Ans: Ht of the pole is 199.301 feet.

103. A man wishes to find out the breadth of a river. From a point of one bank he observes the angle of elevation of a high bldg on the opposite bank to be $34^{\circ}-30'-55''$. He then walk 145 metre away from river to a point in the same line as the previous position and the bldg he has observed. He finds that the angle of elevation of the bldg is now $20^{\circ}-55'-57''$. What was the breadth of the river.

Ans: Breadth of the river is 181.755m.

104. Co-ord of P by E:18670 N: 38850 HT: 610'. Tgt to P by bg= $45^{\circ}-30'$, dist = 4750 yds A/S= $1^{\circ}-50'$. Find out the co-ords of tgt by cal.

Ans: E 15282.06 N-35520.68 Ht-153.88

105. ABC is a triangle side AC is 303.1 yds, AB is 261.4 yds and $\angle ABC$ is $71^{\circ}-41'-00''$. Find out side BC and $\angle BAC$.

Ans: Side BC = 256.184 and , $\angle BAC = 53-21-33'.03''$.

106. The leng of side BC of a triangle is 14.5 M, $\angle ABC = 71^{\circ}$, $\angle ACB = 57^{\circ}$. Cal the length of side AC and AB.

Ans: BC = 17.40, AB =15.43 .

107. The angle of elevation of the top of a mountain from the bottom of tower 180m high angle is $26^{\circ}-25'-00''$. From the top of the tower the angle of elevation is $25-18'-00''$. Find at the height of the mountain.

Ans: Ht of the mountain is 14640.824

108. A ball is thrown from the roof of a bldg with speed of 20 ft/sec at angle 15° below the horizontal . Find the vertical and horizontal components of velocity.

Ans: Horizontal = $20 \times \cos 15 = 19.314$
Vertical = $20 \times \sin 15 = 5.176$

109. A pole of 45' broken at a height of 15' from the ground. Find out the acute angle between the two parts of the pole.

Ans: Angle = 60°

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110. Co-ord of Q bty E:112050 N: 47660 HT: 100'. Q bty to tgt bg= $150^{\circ} - 30'$, dist = 7340 yds A/S= $1^{\circ} - 45'$. Find out the co-ord of tgt by math table.

Ans: E 15664.388 N-41271.589 Ht-772.771